outline

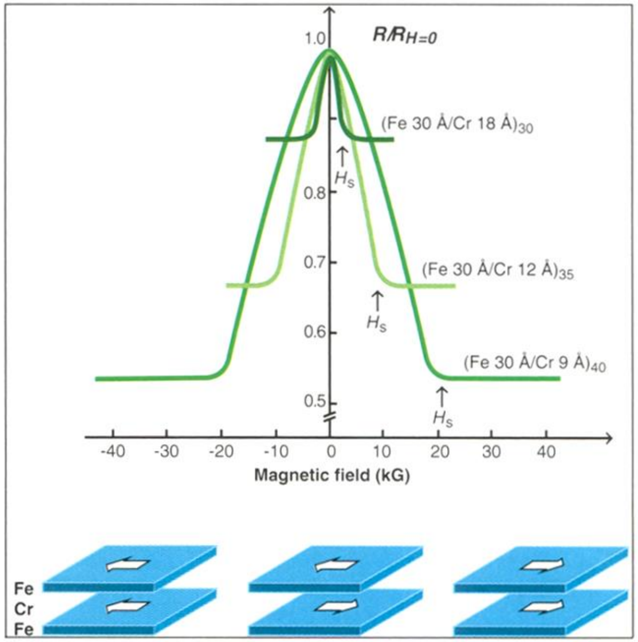
- Giant Magnetoresistance, Tunneling Magnetoresistance
- Spin Transfer Torque
- Pure Spin current (no net charge current)
  - Spin Hall, Inverse Spin Hall effects
  - Spin Pumping effect
  - Spin Seebeck effect
- Micro and nano Magnetics

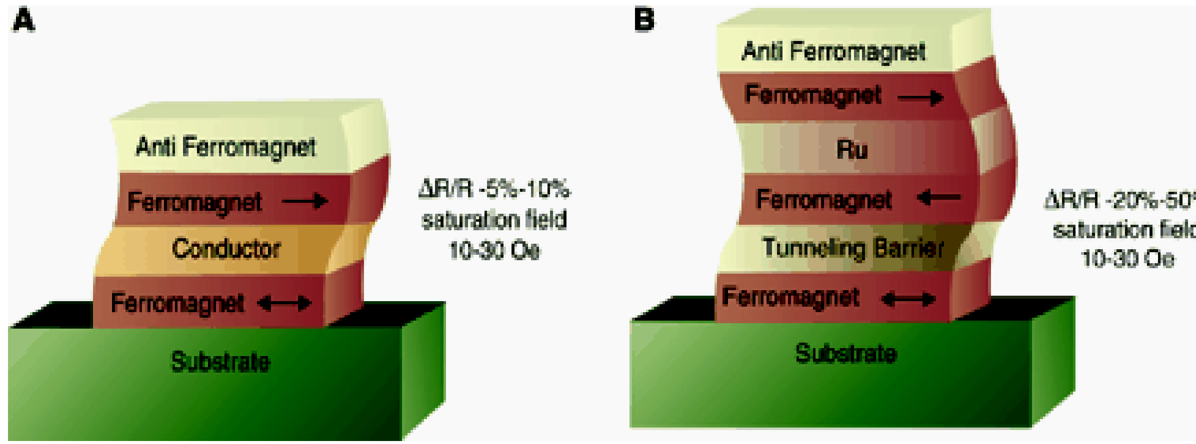
2007 Nobel prize in Physics

2007年諾貝爾物理獎得主 左 亞伯・費爾(Albert Fert) 與
右彼得・葛倫貝格(Peter Grünberg)

(圖片資料來源：Copyright © Nobel Web AB 2007/ Photo: Hans Mehlin)

Giant Magnetoresistance
Tunneling Magnetoresistance





Discovery of Giant MR -- Two-current model combines with magnetic coupling in multilayers

Spin-dependent transport structures. (A) Spin valve. (B) Magnetic tunnel junction. (from Science)

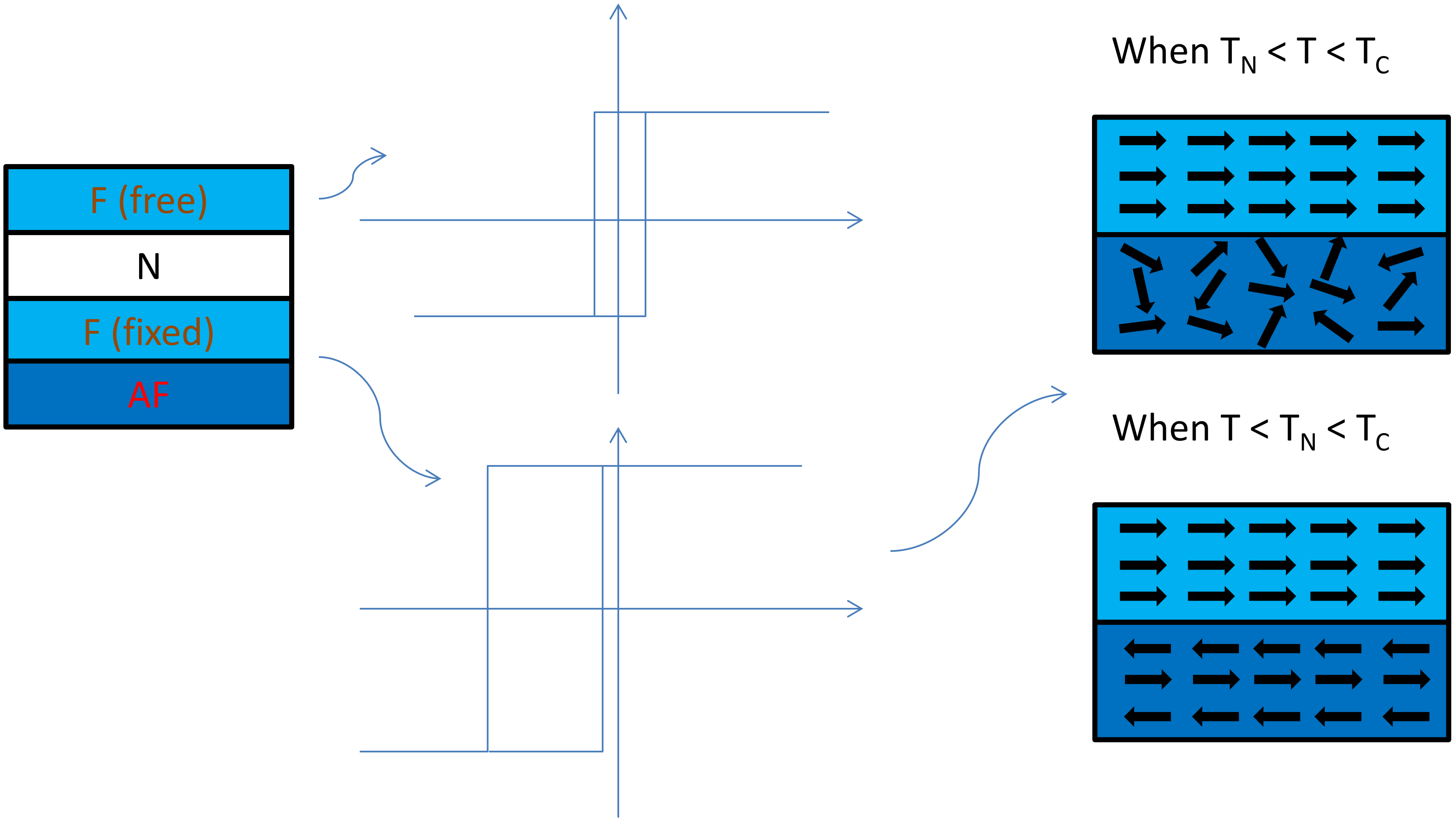
Moodera's group, PRL **74**, 3273 (1995)

Fert's group, PRL **61**, 2472 (1988)

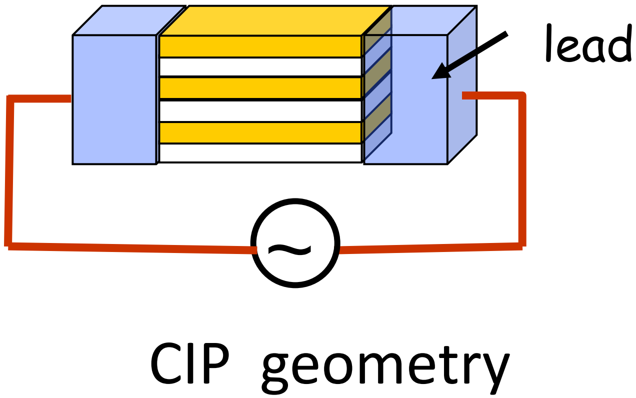
Miyazaki's group, JMMM **139**, L231(1995)

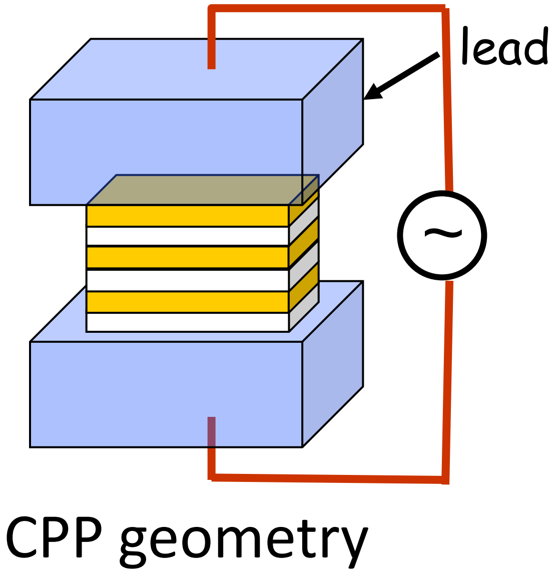
Spin valve –

a sandwich structure with a free ferromagnetic layer (F) and a fixed F layer pinned by an antiferromagnetic (AF) layer



Transport geometry





- In metallic multilayers, CIP resistance can be measured easily, CPP resistance needs special techniques.
- From CPP resistance in metallic multilayers, one can measure interface resistances, spin diffusion lengths, and polarization in ferromagnetic materials, etc.
- CPP magnetoresistance of magnetic multilayers: A critical review

  Jack Bass

  Journal of Magnetism and Magnetic Materials 408 (2016) 244–320

Valet and Fert model of (CPP-)GMR

Based on the Boltzmann equation

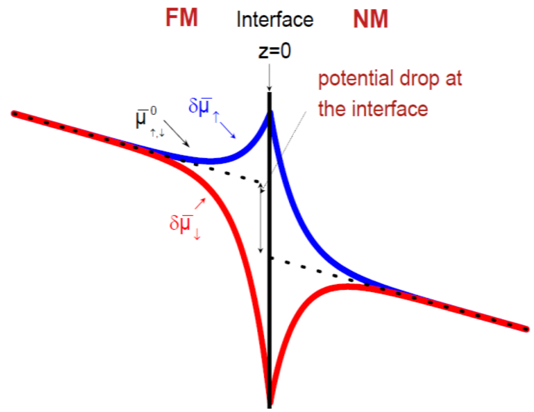
A semi-classical model with spin taken into consideration

$$j_{+(-)} = \frac{1}{e\rho_{+(-)}} \frac{\partial \mu_{+(-)}}{\partial x}$$

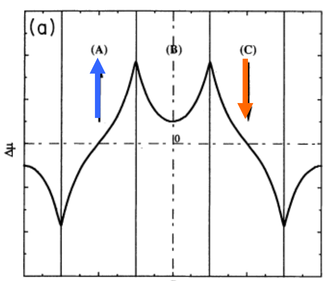
$$j_+ + j_- = j_e$$

$$\frac{\partial (j_+ - j_-)}{\partial x} = \frac{2eN(E_F)\Delta\mu}{\tau_{sf}}$$

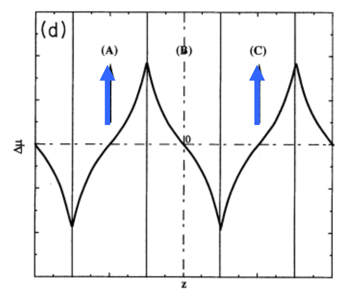
$$\frac{\partial^2 \mu_{+(-)}}{\partial z^2} = \frac{\mu_{+(-)}}{l_{sf}^2} \qquad l_{sf}^F = \left[\lambda_{sf}^F / 3(\lambda_\uparrow^{-1} + \lambda_\downarrow^{-1})\right]^{1/2}, \quad l_{sf}^N = \left[\lambda_{sf}^N \lambda / 6\right]^{1/2}$$



Δμ for antiparallel aligned multilayers



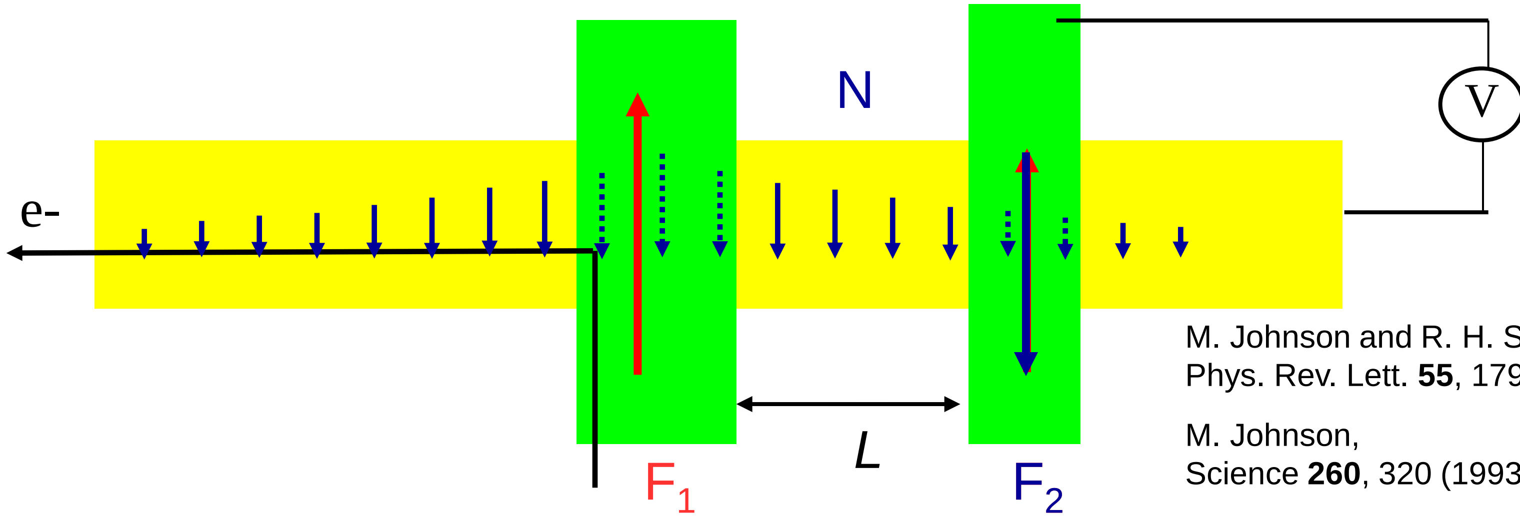
Δμ for parallel aligned multilayers



Spin imbalance induced charge accumulation at the interface is important

Spin diffusion length, instead of mean free path, is the dominant physical length scale

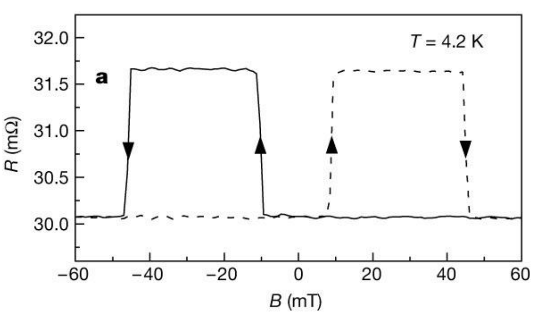
Spin Diffusion: The Johnson Transistor
non-local measurement

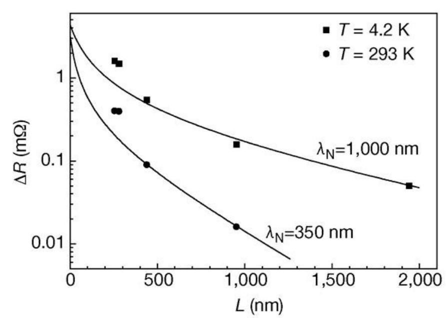


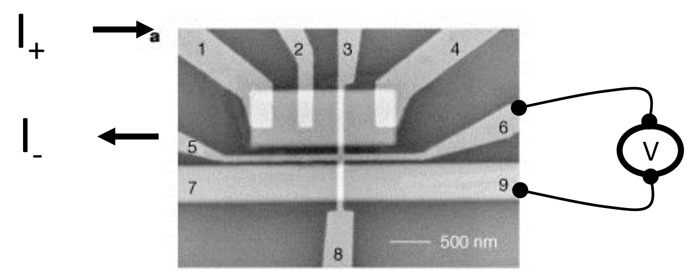
M. Johnson and R. H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985)

M. Johnson, Science **260**, 320 (1993)

First Experimental Demonstrations







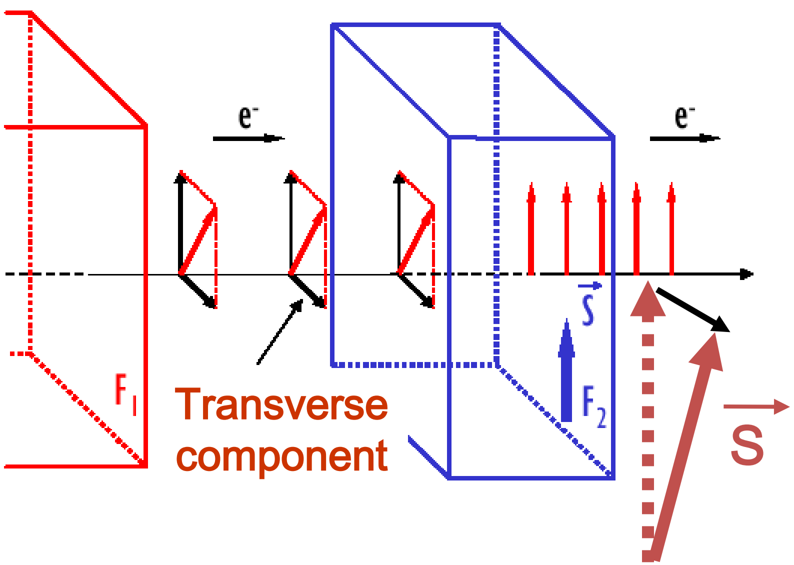
Cu film: λ_s = 1 μm (4.2 K)

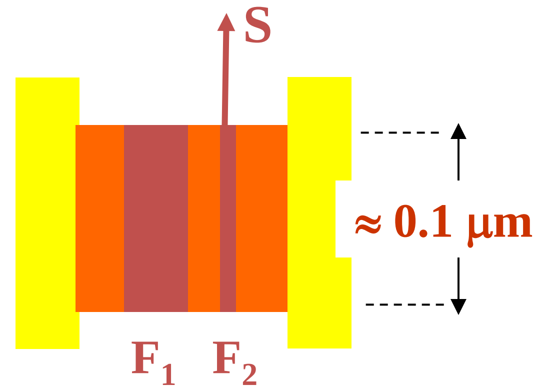
Jedema *et al.*, Nature **410**, 345 (2001)

outline

- Giant Magnetoresistance, Tunneling Magnetoresistance
- Spin Transfer Torque
- Pure Spin current (no net charge current)
    - Spin Hall, Inverse Spin Hall effects
    - Spin Pumping effect
    - Spin Seebeck effect
- Micro and nano Magnetics

Spin Transfer Torque





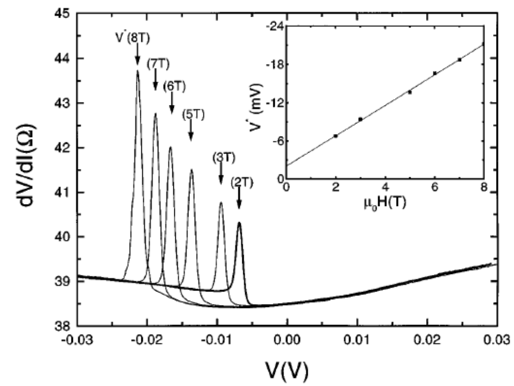


FIG. 1. The point contact $dV/dI(V)$ spectra for a series of magnetic fields (2, 3, 5, 6, 7, and 8 T) revealing an upward step and a corresponding peak in dV/dI at a certain negative bias voltage $V^*(H)$. The inset shows that $V^*(H)$ increases linearly with the applied magnetic field H.

Tsoi et al. PRL **61**, 2472 (1998)

The transverse spin component is lost by the conduction electrons, transferred to the global spin of the layer $\vec{S}$

$$\dot{S}_{1,2} = (I_e g/e)\hat{s}_{1,2}\times(\hat{s}_1\times\hat{s}_2)$$

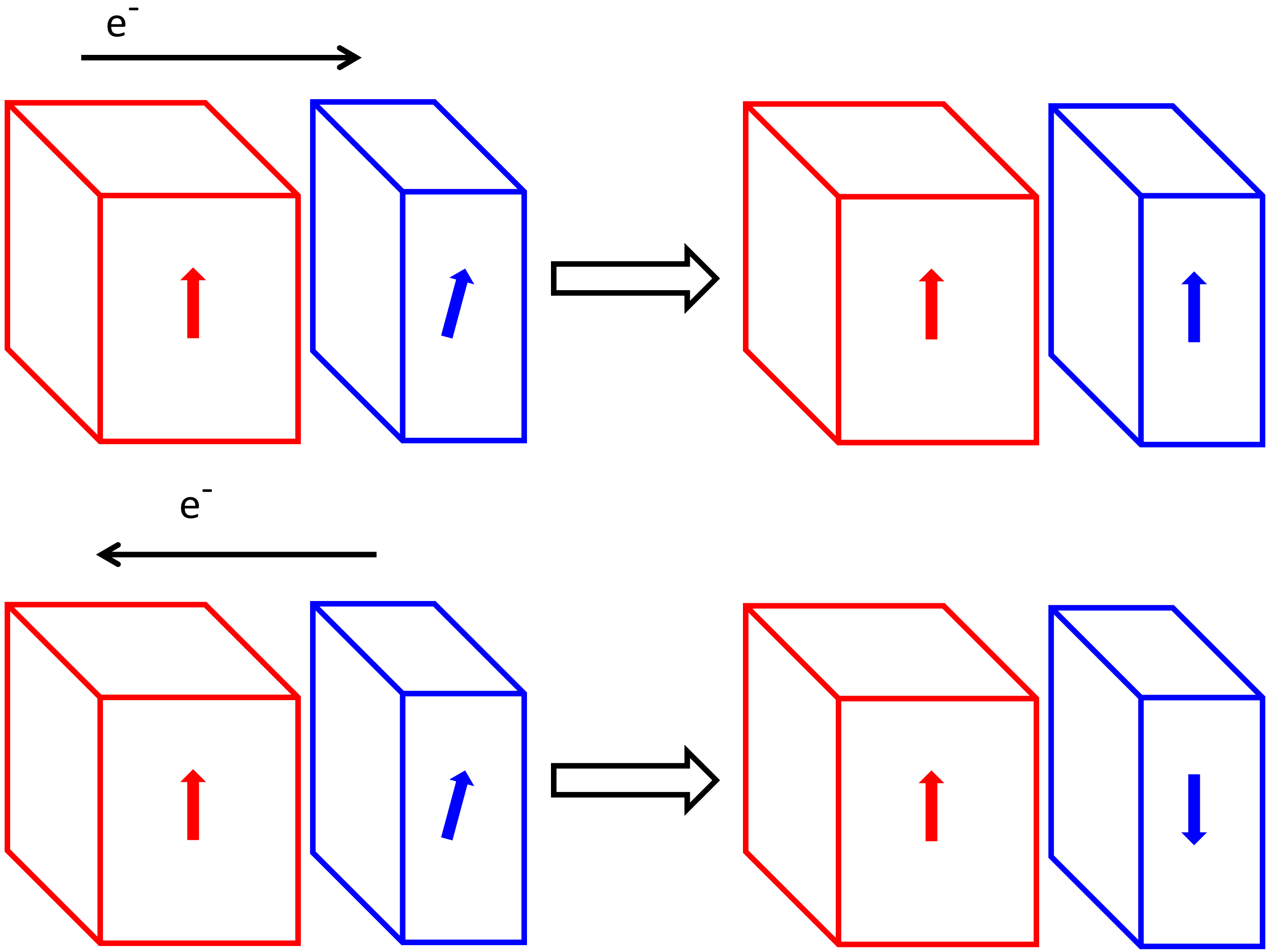
Slonczewski JMMM **159**, L1 (1996)

Modified Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{dm}{dt} = -\gamma m\times H_{eff} + \alpha m\times\frac{dm}{dt} + \frac{\gamma\hbar PI}{2e\mu_0 M_S V}(m\times\sigma\times m)$$

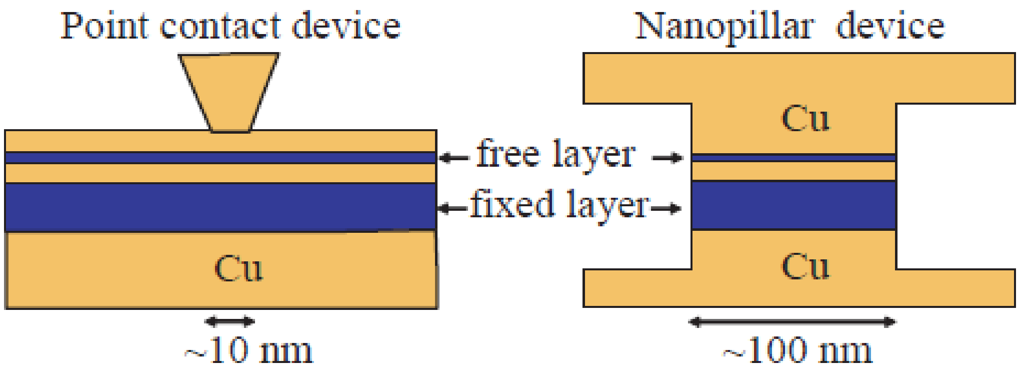
Experimantally determined current density ~10^{10}-10^{12}A/m^2

Spin Transfer Torque

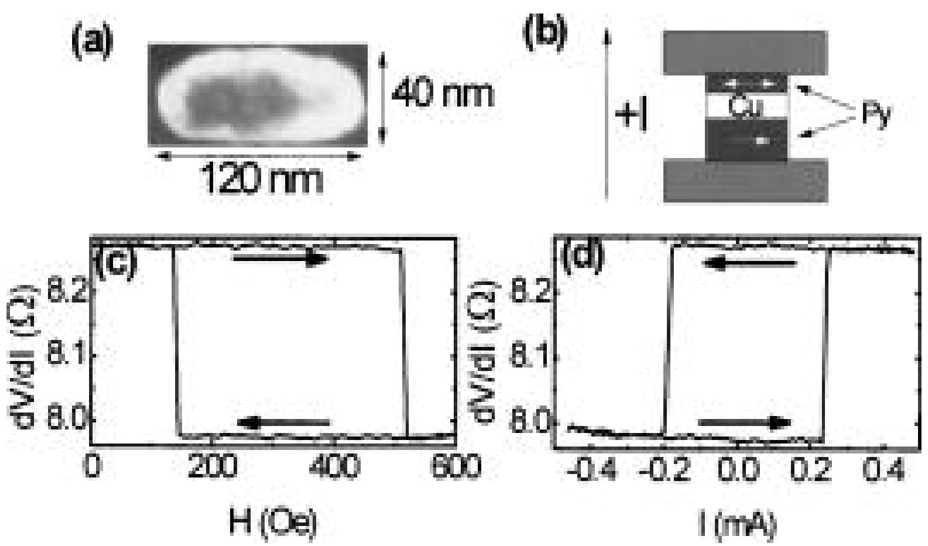


In a trilayer, current direction determines the relative orientation of F_1 and F_2

Spin Transfer Torque



Ralph and Stiles "Spin transfer torques". *JMMM* **320,** 1190–1216 (2008).



(c) Minor loop of free layer and (d) spin transfer curve at 293*K*
120 Cu/20 Py/12 Cu/X Py/2 Cu/30 Au

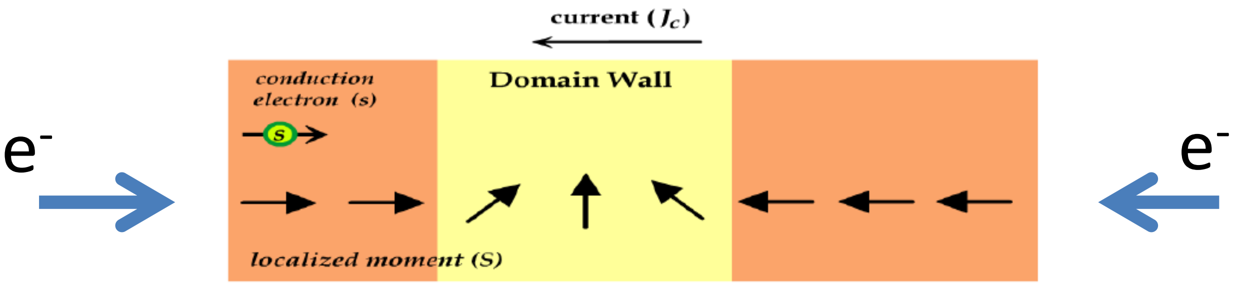
Ralph and Buhrman's group, APL **87**, 112507 (2005)

Spin Transfer Torque

Landau-Lifshitz-Gilbert equation with Spin Transfer Torque terms

Current induced domain wall motion

Passing spin polarized current from Domain A to Domain B⇒B switches



Domain A

Domain B

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H}_{eff} + \frac{\alpha}{M_s} \vec{M} \times \frac{\partial \vec{M}}{\partial t} + \vec{\mathrm{T}}_{STT}$$

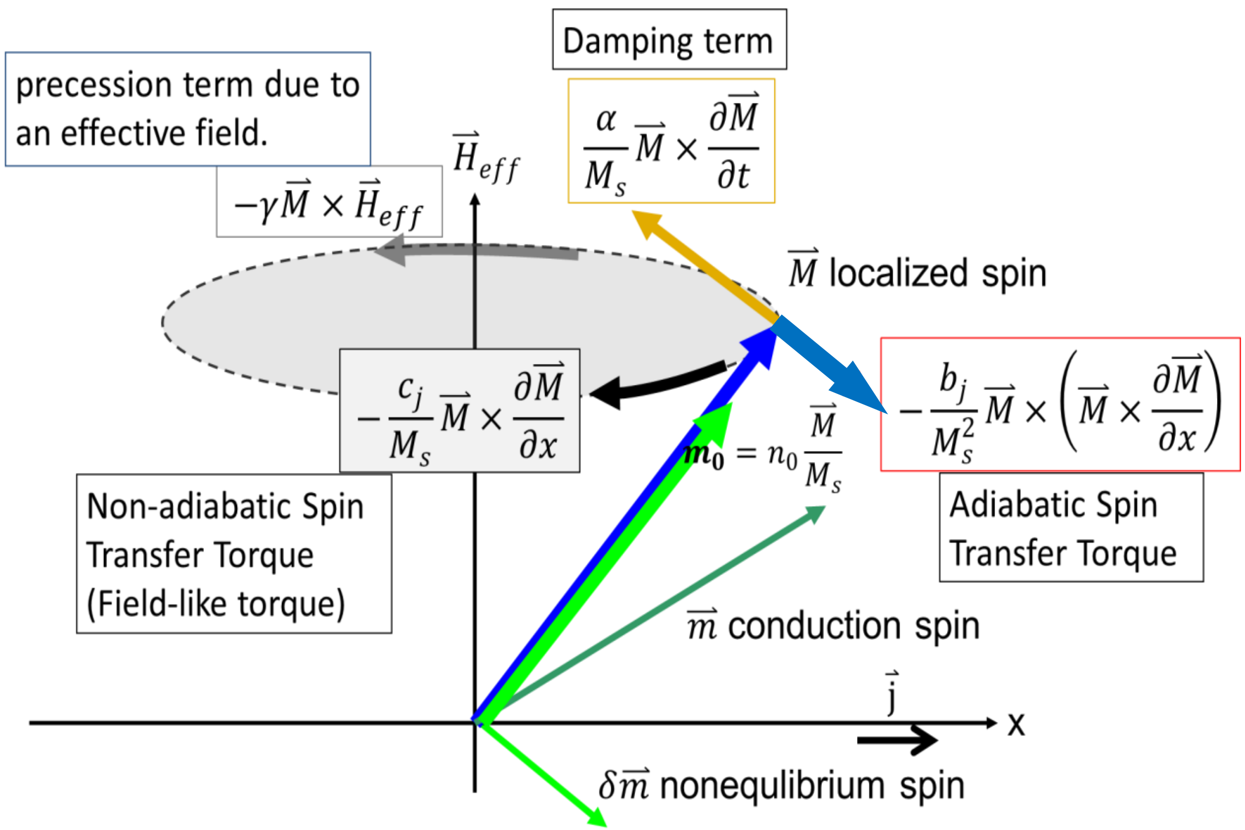
Berger, *JAP* **55**, 1954 (1984)
Tatara *et. al.*, *PRL* **92**, 086601 (2004)
Zhang *et. al.*, *PRL* **93**, 127204 (2004)
Thiaville *et. al.*, *Europhys. Lett*. **69**, 990 (2005)
Stiles *et. al.*, *PRB* **75**, 214423 (2007)

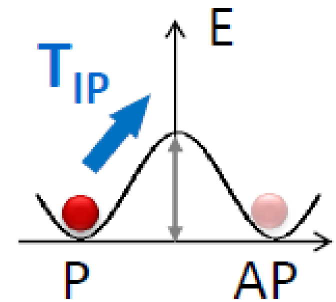
Spin Transfer Torque

Landau-Lifshitz-Gilbert equation with Spin Transfer Torque terms

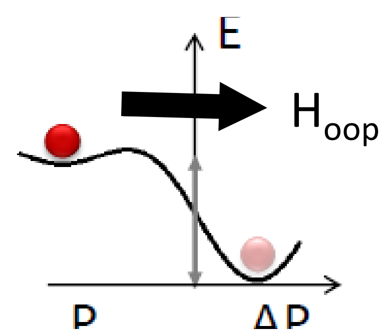
$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H}_{\text{eff}} + \frac{\alpha}{M_s} \vec{M} \times \frac{\partial \vec{M}}{\partial t} - \frac{b_j}{M_s^2} \vec{M} \times \left(\vec{M} \times \frac{\partial \vec{M}}{\partial x}\right) - \frac{c_j}{M_s} \vec{M} \times \frac{\partial \vec{M}}{\partial x}$$

$b_j, c_j \sim JP/t$





In-plane torque
Anti-damping
Destabilizes M



Out-of-plane torque
Field-like torque
Modifies energy barrier

Onsager reciprocity relations

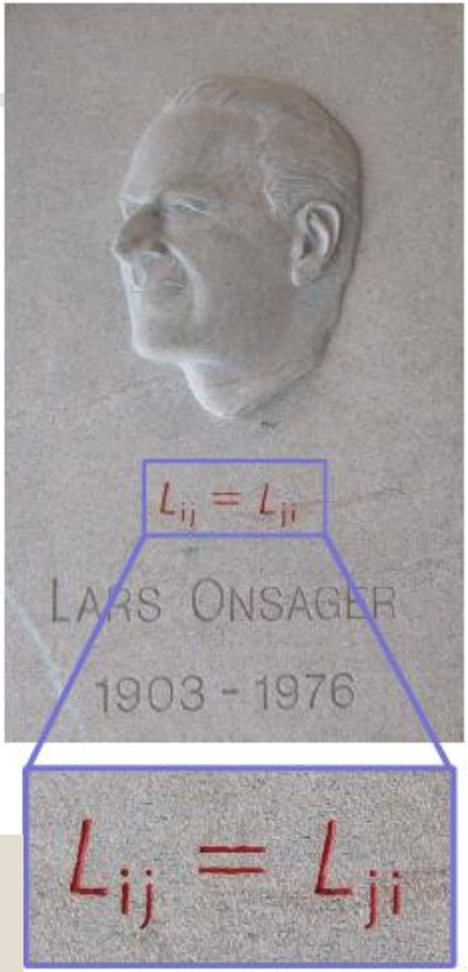
Conjugate variables

X_i	generalized forces
J_i	generalized currents
$J_i = \sum_j L_{ij} X_j$	linear response

i = {mass, charge, spin, energy, ...}

$\dot{S} = \sum_i X_i J_i$ entropy creation rate

$$L_{ij}(\mathbf{m}, \mathbf{H}_{ext}) = \varepsilon_i \varepsilon_j L_{ji}(-\mathbf{m}, -\mathbf{H}_{ext})$$



Equality between certain relations between flows and forces out of equilibrium

Currents can induce magnetization excitations

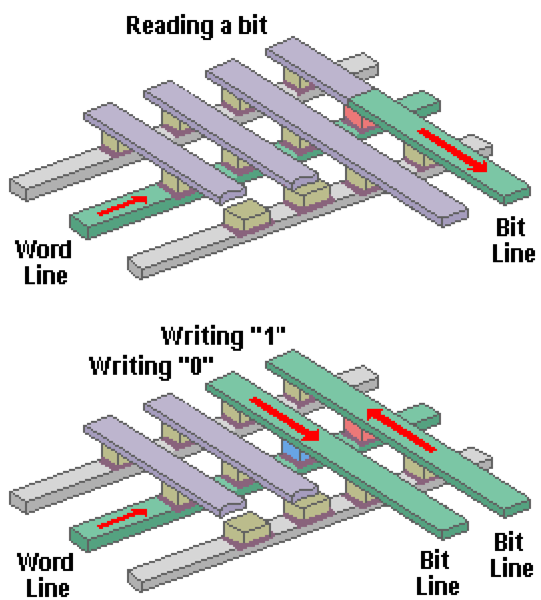
↕

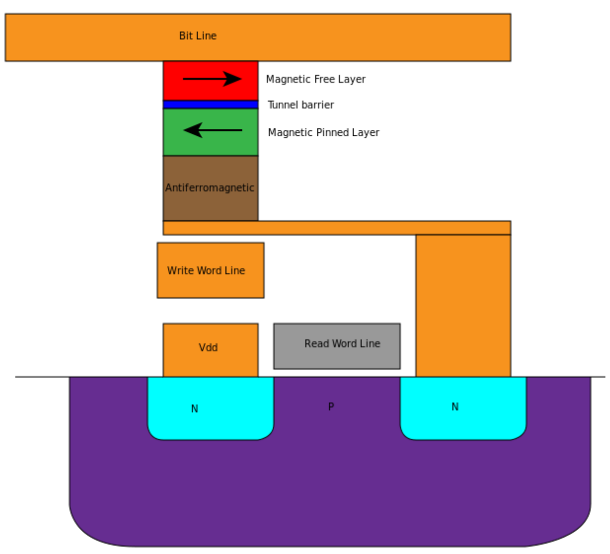
A time-dependent magnetization can induce (charge and spin) currents

Industrial applications

Read head in hard drives

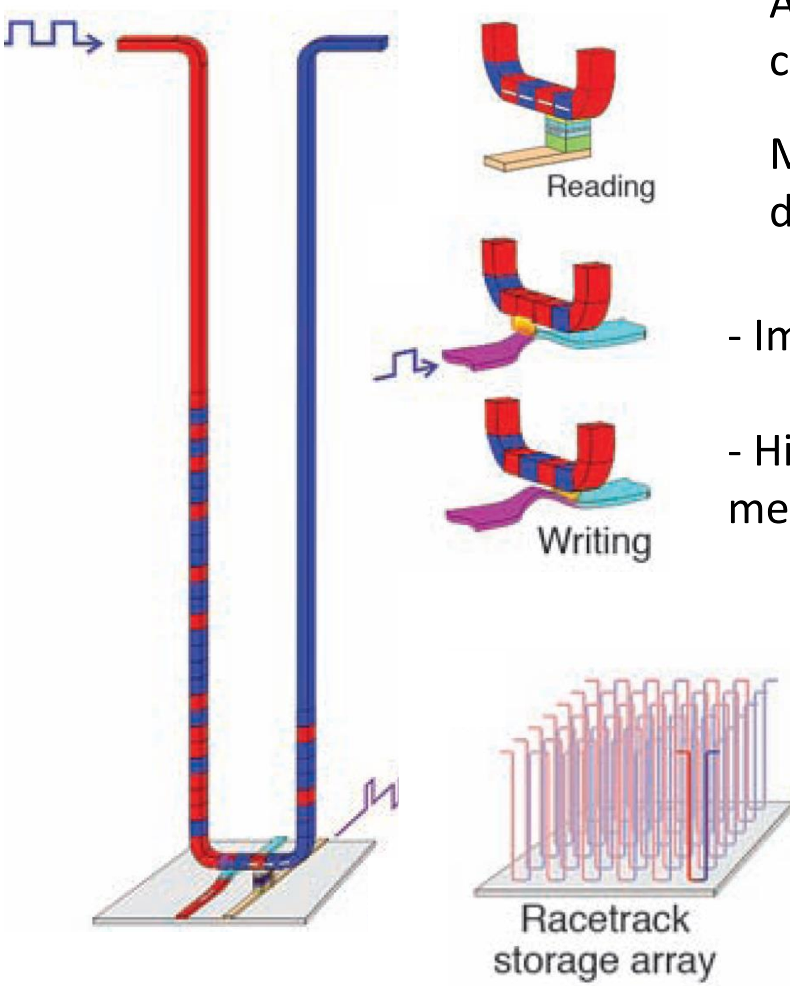
MRAM





Application of Spin Transfer Torque

Magnetic Domain-Wall Racetrack Memory



Dr. Stuart S. P. Parkin Science **320**, 190 (2008)

A novel three-dimensional spintronic storage class memory

Magnetic nanowires: information stored in the domain walls

- Immense storage capacity of a hard disk drive

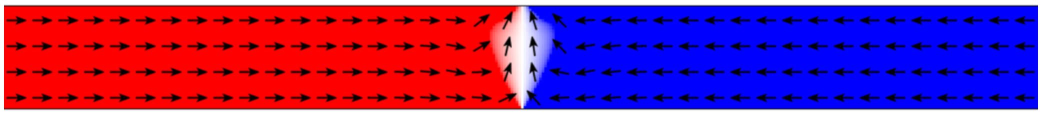
- High reliability and performance of solid state memory (DRAM, FLASH, SRAM...)

→ **Understanding of current induced domain wall (DW) motion**

Application of Spin Transfer Torque

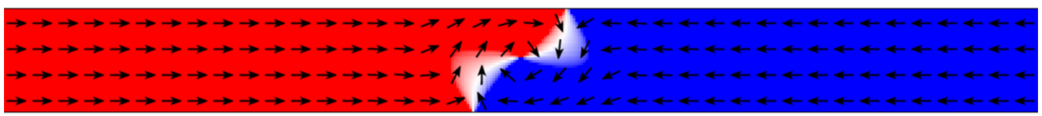
Domain Wall Structures in Permalloy Nanowires

(a)

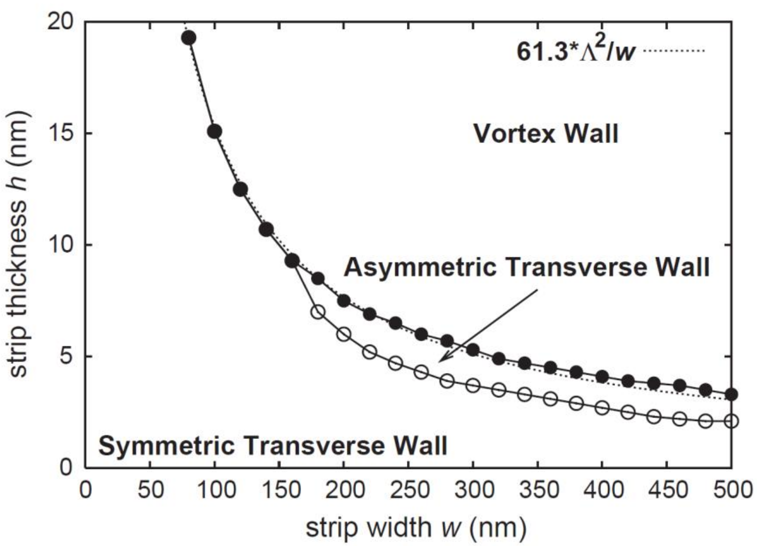


Transvers DW

(b)



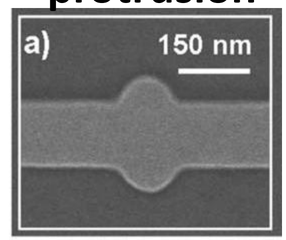
Vortex DW



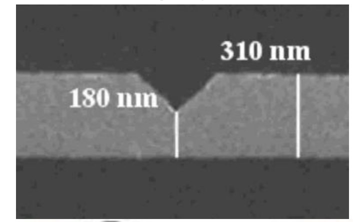
J. Magn. Magn. Mater. **290**, 750 (2005)

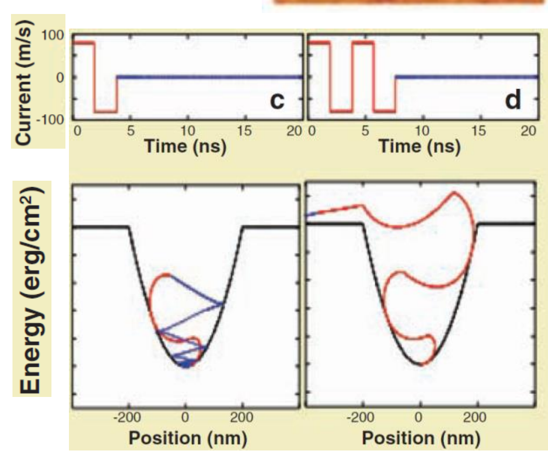
DW traps

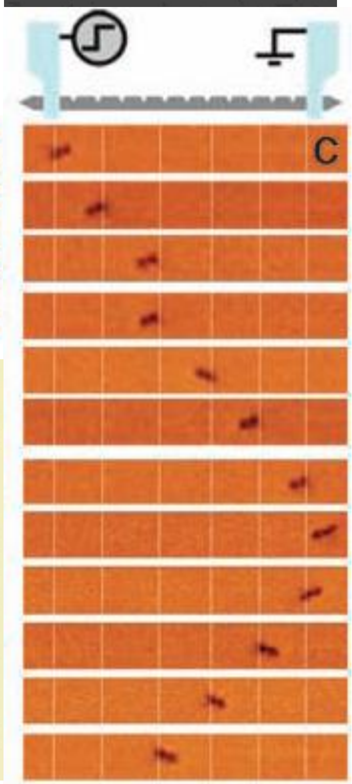
protrusion



notch





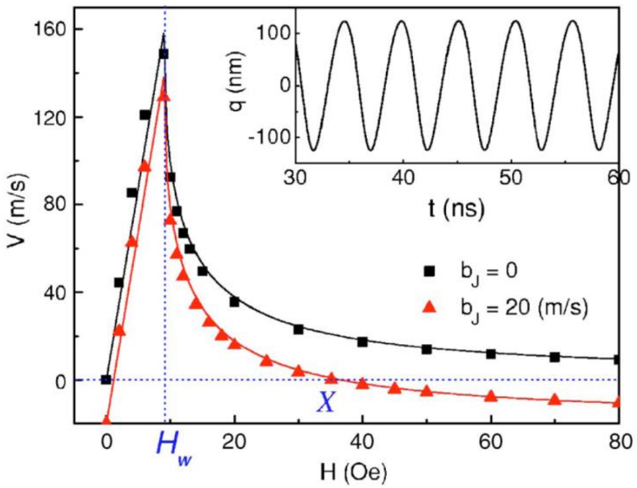


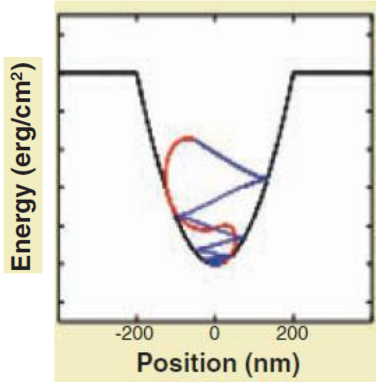
Application of Spin Transfer Torque

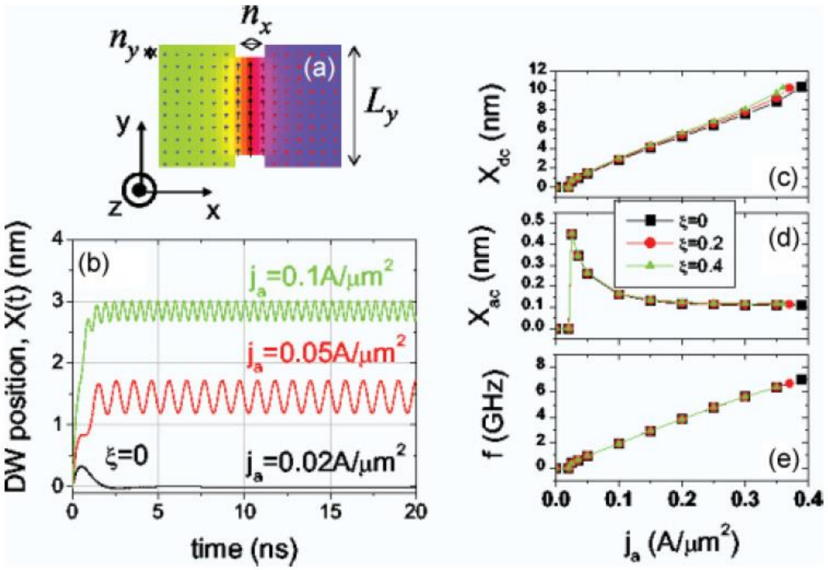
DW Oscillators

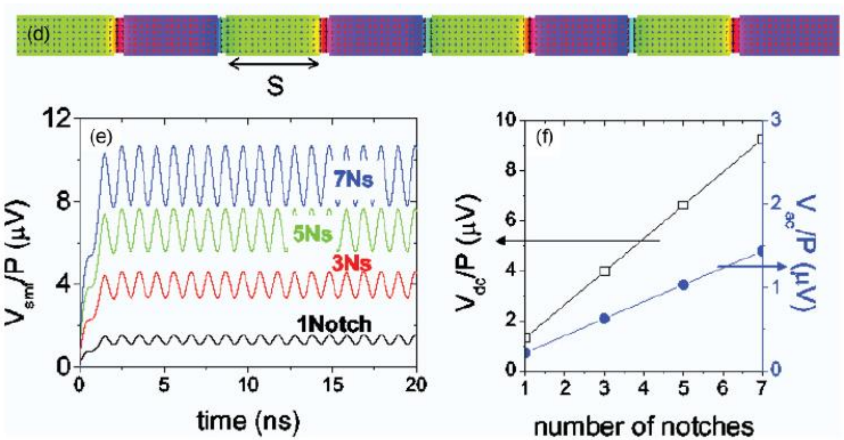
$j_W < j_a < j_{dep}$

Walker breackdown





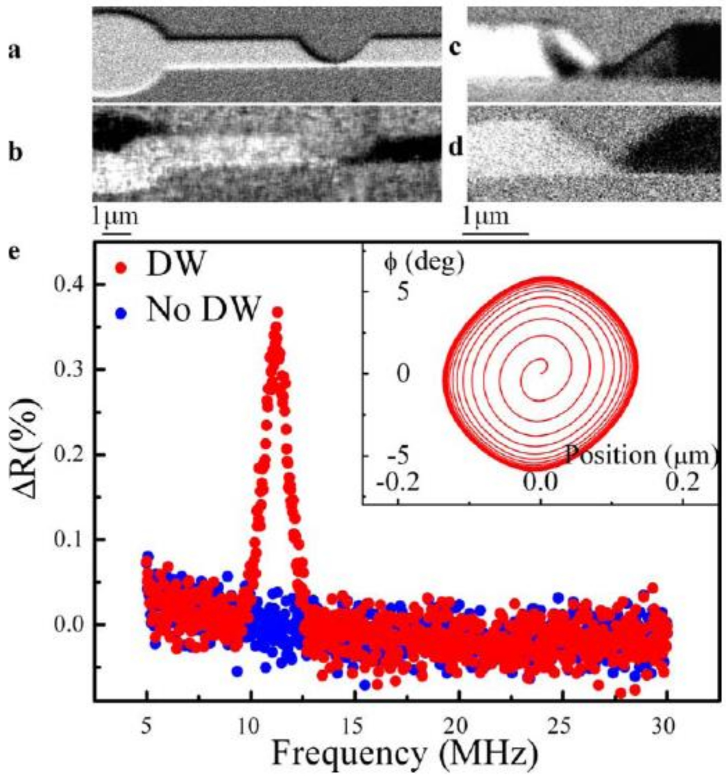


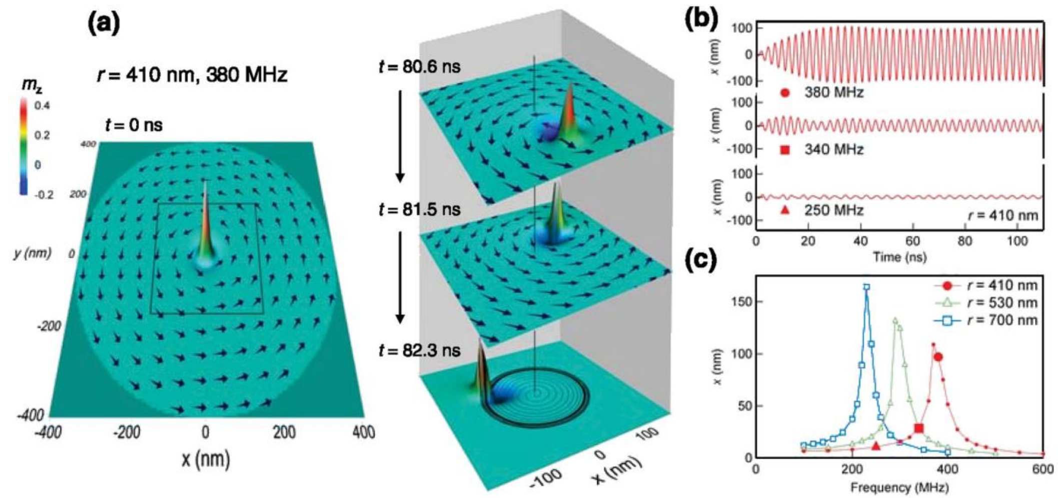


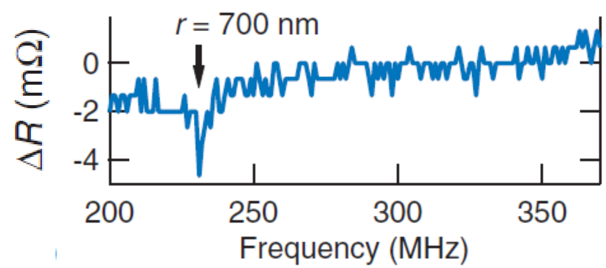
PHYSICAL REVIEW B **83**, 174444 (2011)
Appl. Phys. Lett. **90**, 142508 (2007)

Application of Spin Transfer Torque

AC Current-Induced DW Resonance







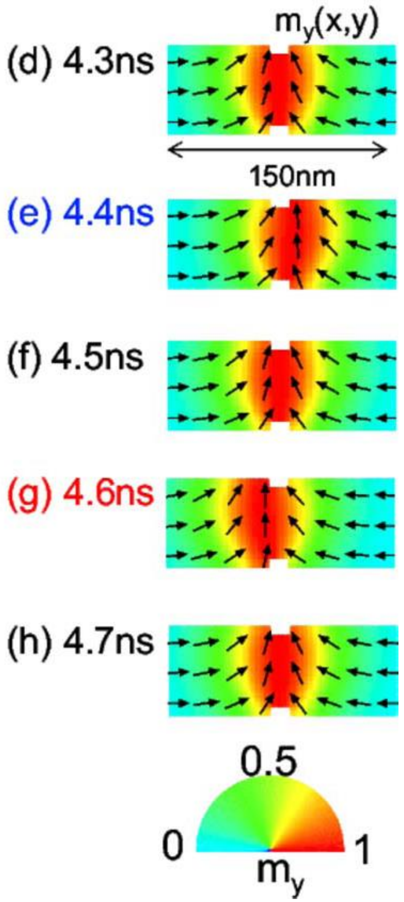
PRB **81**, 060402 (2010), PRL **97**, 107204 (2006)

Application of Spin Transfer Torque

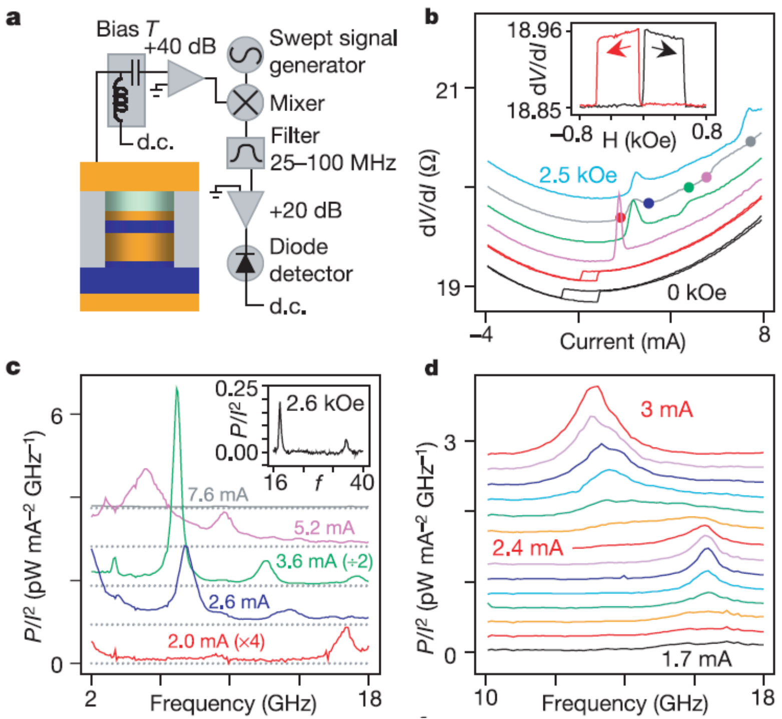
Radio-Frequency DW Oscillators

AC

DC

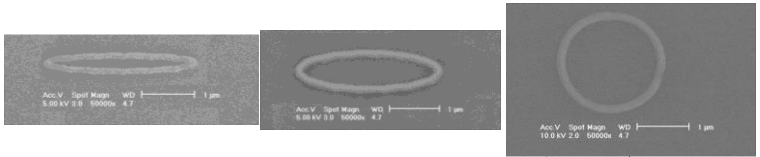


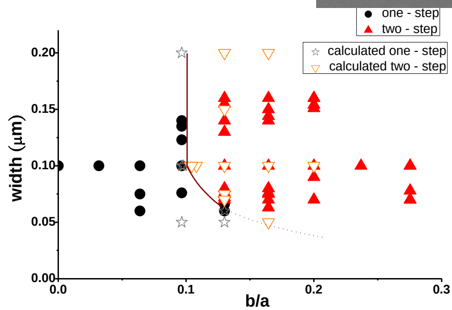
- CPP-nanopillar

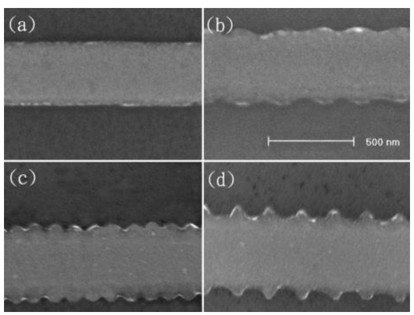


Nature **425**, 380 (2003)

Our works







Magnetic nanostructures

- "Quantitative analysis of magnetization reversal in submicron S-patterned structures with narrow constrictions by magnetic force microscopy". APL **86**, 053111 (2005).
- "Observation of Room Temperature Ferromagnetic Behavior in Cluster Free, Co doped HfO_2 Films". APL **91**, 082504 (2007).
- "Variation of magnetization reversal in pseudo-spin-valve elliptical rings". APL **94**, 233103 (2009).
- "Compensation between magnetoresistance and switching current in Co/Cu/Co spin valve pillar structure". APL **96**, 093110 (2010).
- "Exchange bias in spin glass (FeAu)/NiFe thin films". APL **96**, 162502 (2010).
- "Demonstration of edge roughness effect on the magnetization reversal of spin valve submicron wires". APL **97**, 022109 (2010).
- "Current induced localized domain wall oscillators in NiFe/Cu/NiFe submicron wires". APL **101**, 242404 (2012).
- "Inverse spin Hall effect induced by spin pumping into semiconducting ZnO". APL **104**, 052401 (2014).

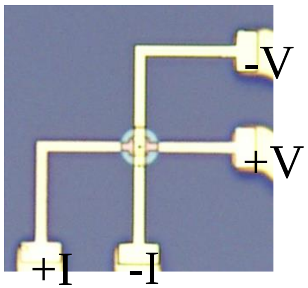
outline

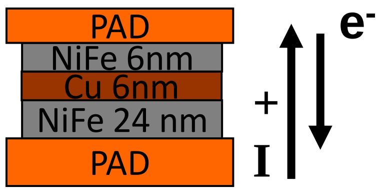
- Giant Magnetoresistance, Tunneling Magnetoresistance
- Pure Spin current (no net charge current)
  - Spin Hall, Inverse Spin Hall effects
  - Spin Pumping effect
  - Spin Seebeck effect
- Spin Transfer Torque
- Micro and nano Magnetics

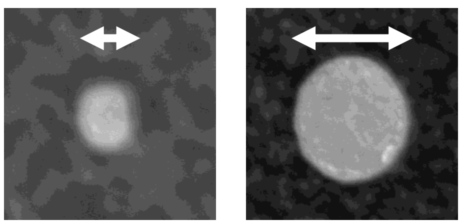
Nano Magnetism

Vortex induced by dc current in a circular magnetic spin valve nanopillar

L. J. Chang and S. F. Lee

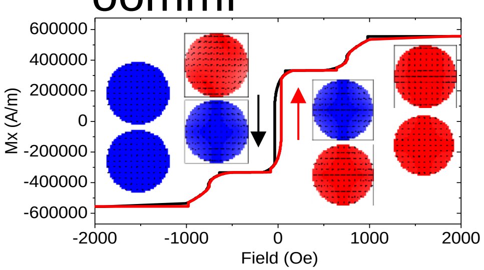




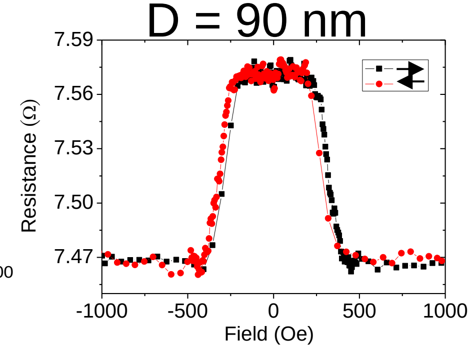


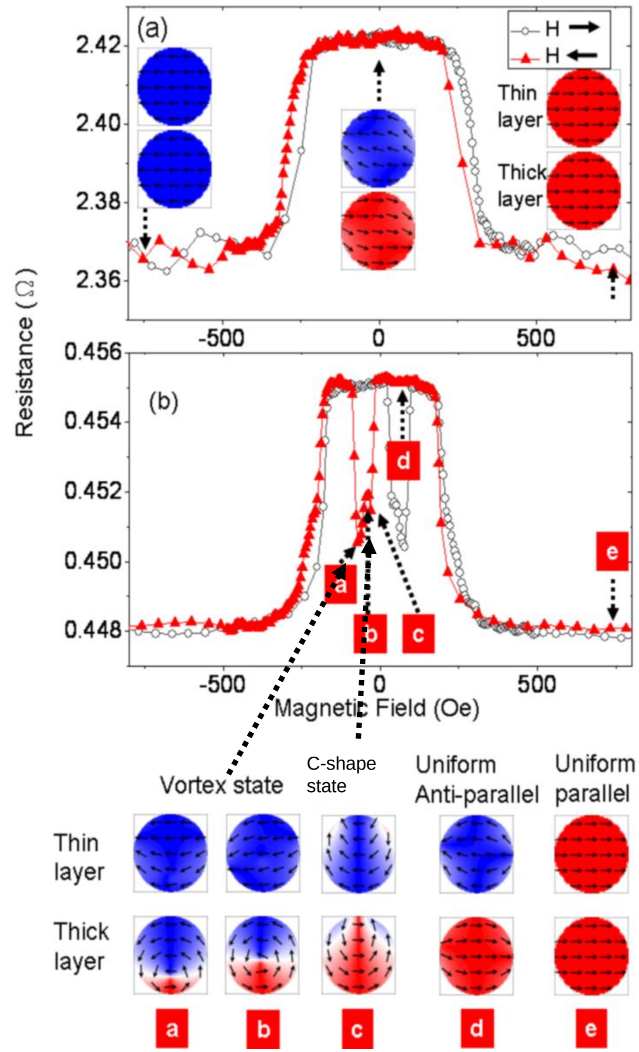
90 nm 380 nm

oommf



D = 90 nm

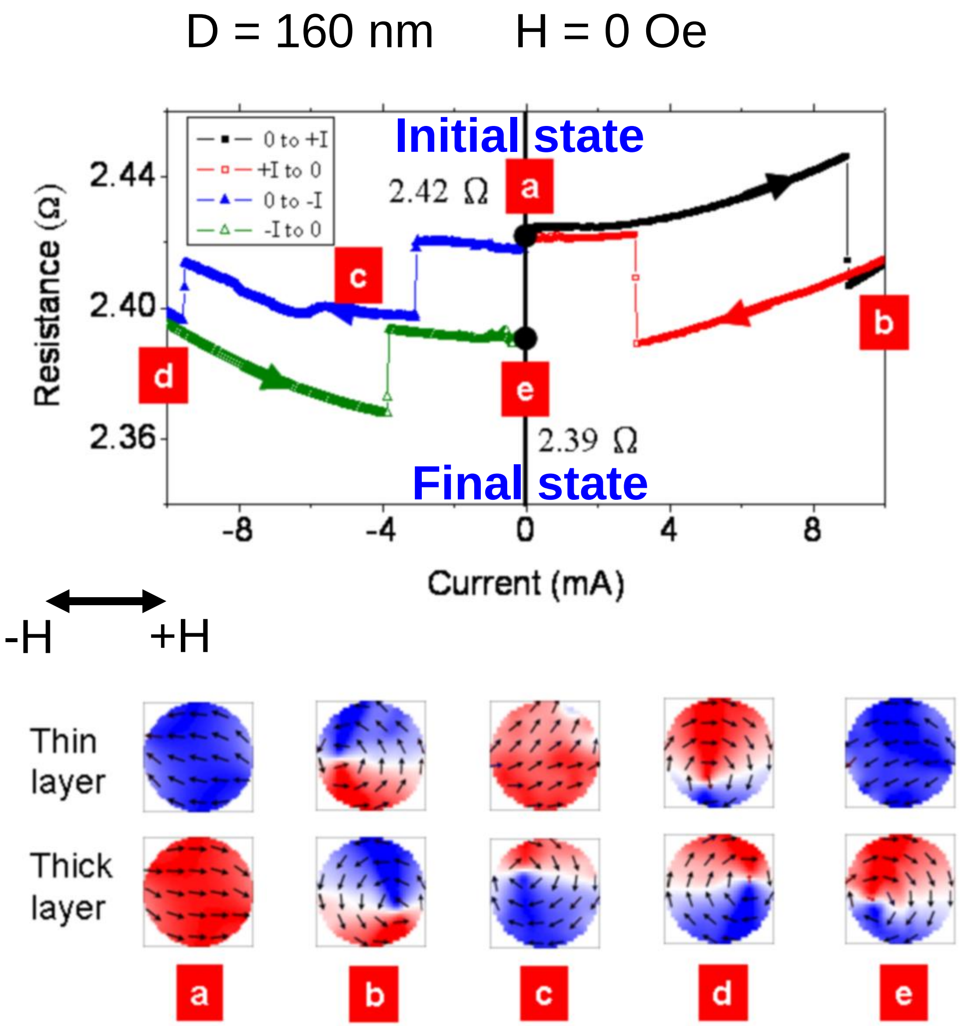




D = 160 nm

D = 380 nm

Current driven vortex nucleation



Other research interest include superconductor-magnetic material proximity effect, Ferromagnetic Resonance etc.

Domain wall oscillation in a trapping potential

Theoretical Backgrounds

Resonant DW induced by AC spin-polarized current in Ferromagnetic strips

DW dynamics equation

$$(1+\alpha^2)m\frac{d^2x}{dt^2} = F_p(x) + F_f + F_s + F_d$$

where $m = \frac{2(\mu_0 L_y L_z)}{\gamma_0^2 (N_z - N_y)\Delta_0}$ is the effective DW mass (kg), and the other variables are listed below.

L_y : width of wire (m)

L_z : thickness of wire (m)

μ_0 : permeability ($4\pi \times 10^{-7}$ $VsA^{-1}m^{-1}$)

γ_0 : electron gyromagnetic ratio (2.2×10^5 $Vs^2m^{-1}kg^{-1}$)

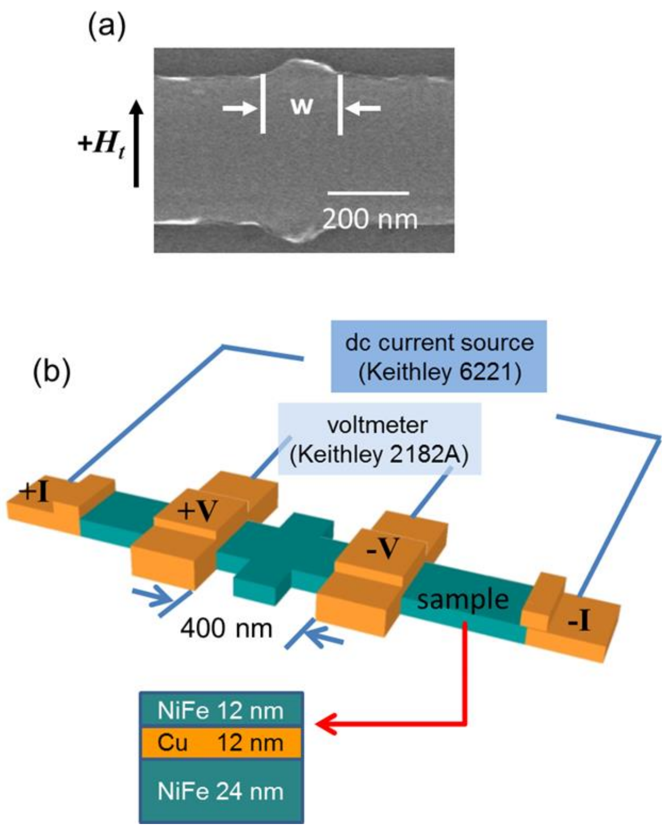
N_z, N_y : transverse demagnetizing factors

Δ_0 : DW width (m)

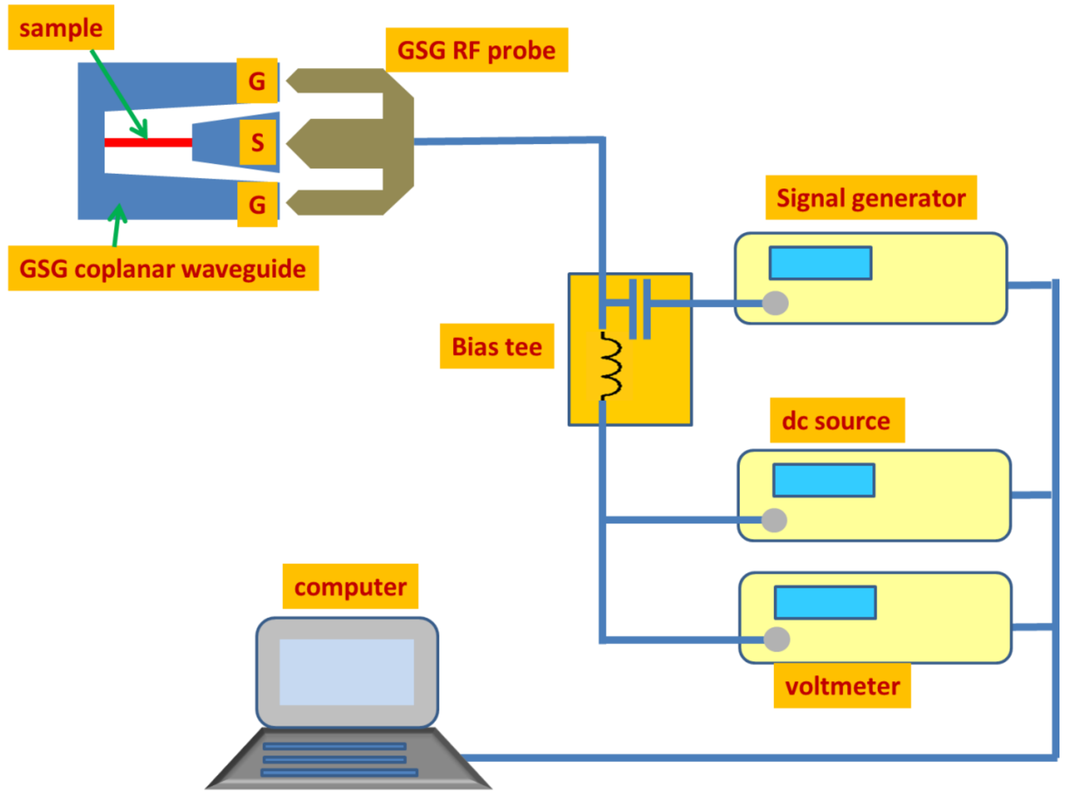
x : DW position (m)

Experiment Methods

four point probe measurement circuit



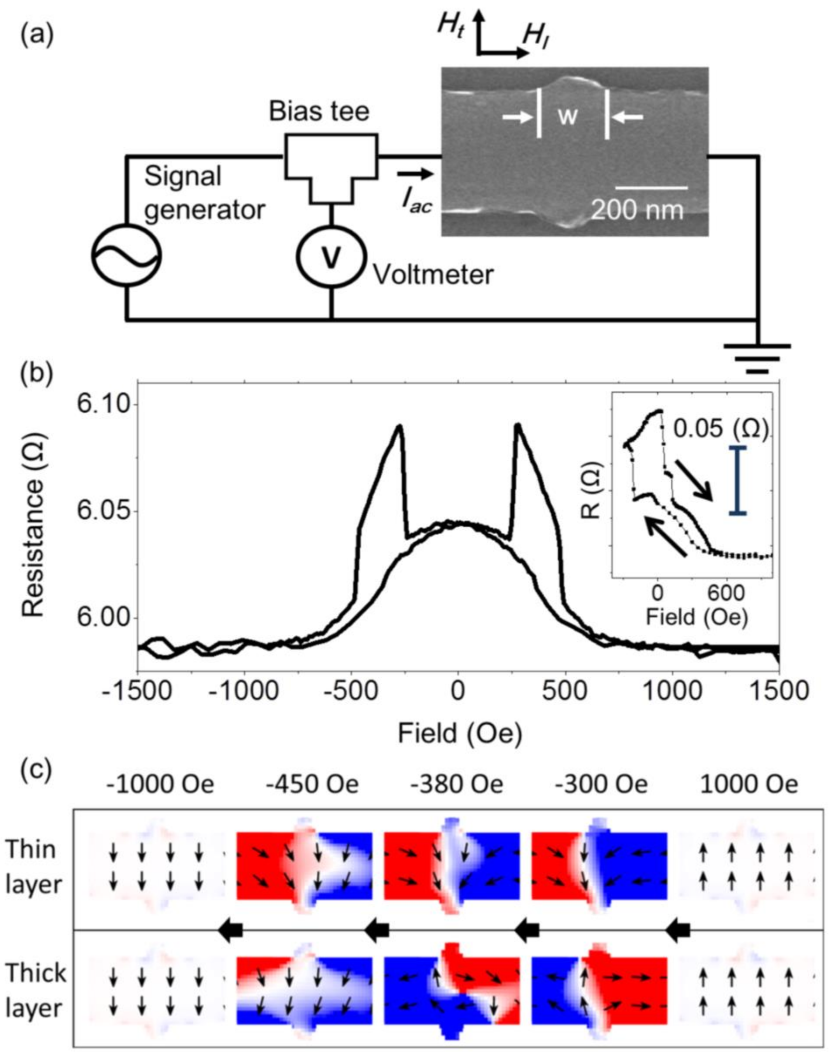
high frequency measurements circuit



Measurement and simulation results

AC current induced localized domain wall oscillators in NiFe/Cu/NiFe submicron wires

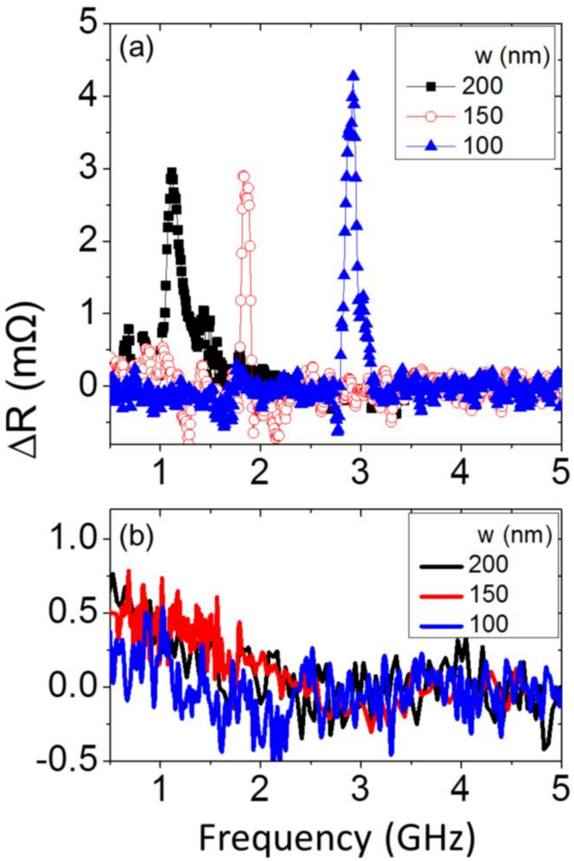
Nucleation of Pinned anti-parallel transverse DW



Measurement and simulation results

AC current induced localized domain wall oscillators in NiFe/Cu/NiFe submicron wires

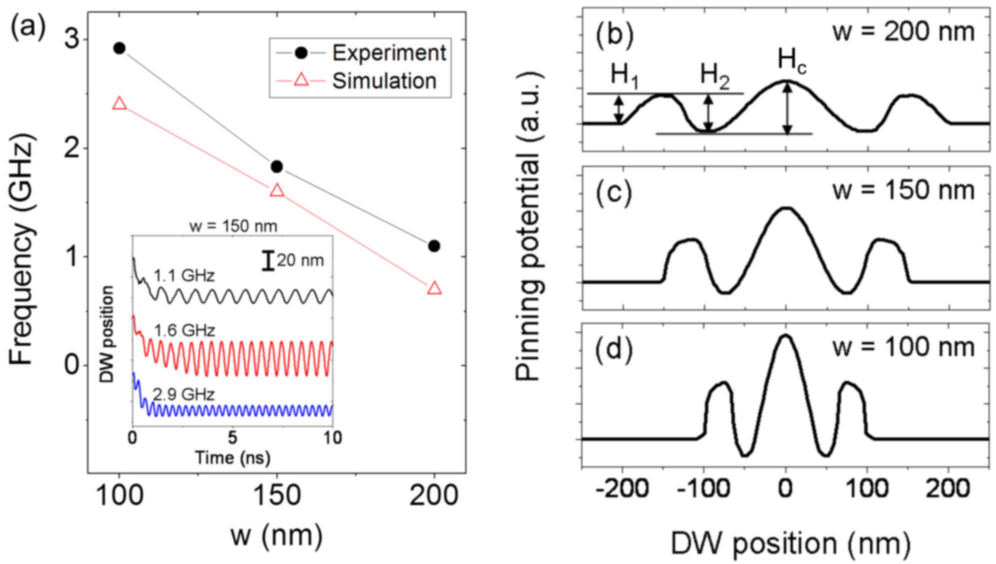
DW resonators for frequency-selective operation



(a) Experimental measurement of the ac current induces resonance excitation of pinned DW trapped at the protrusion. Resistance change as a function of ac excitation current frequency for the submicron wires containing artificial symmetric protrusions with three different widths of protrusion w = 200, 150, and 100 nm. (b) The response curve measured at the saturation field with a uniform state of submicron wires (without DW). The ΔR is observed unchanged with frequency for each of the samples.

Measurement and simulation results

AC current induced localized domain wall oscillators in NiFe/Cu/NiFe submicron wires

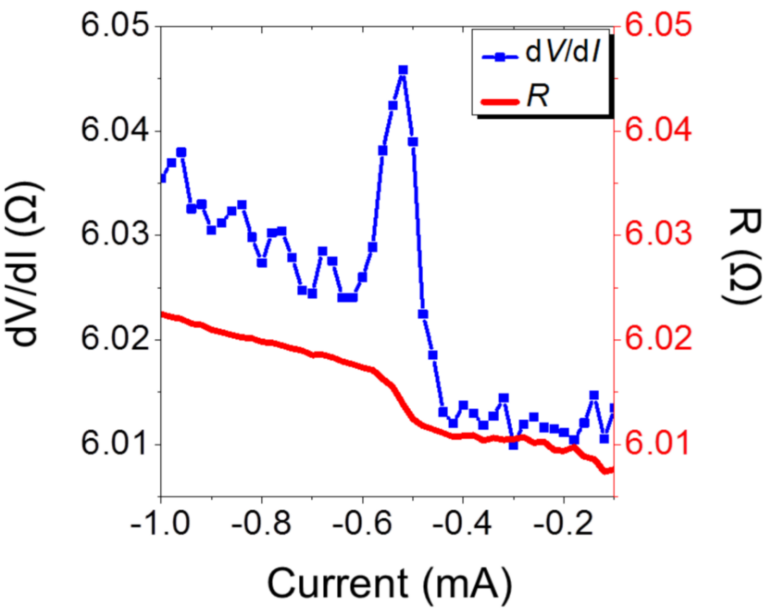


Resonance frequency of pinned DW dependence on the width of trap w, the solid circles and the open triangles indicate the experiment and simulation results respectively. The inset shows the simulated time evolutions of the DW motion with w = 150 nm. (b)-(d) Potential landscape of pinned DW from micromagnetic simulation with three different width of protrusion w = 200, 150, 100 nm.

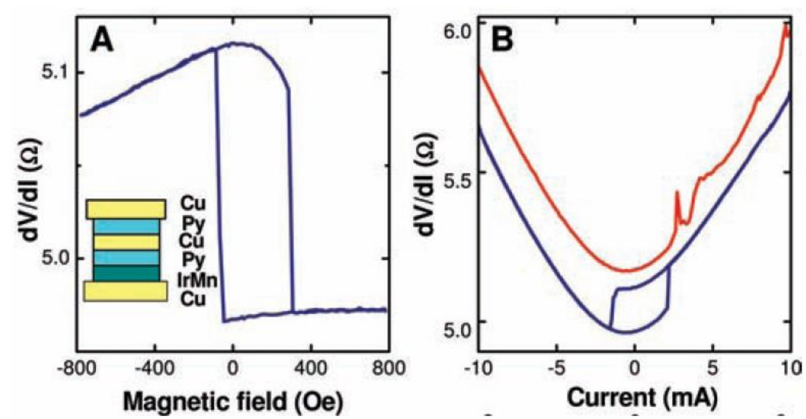
Measurement and simulation results

Reversible domain wall motion induced by dc current in NiFe/Cu/NiFe submicron wires

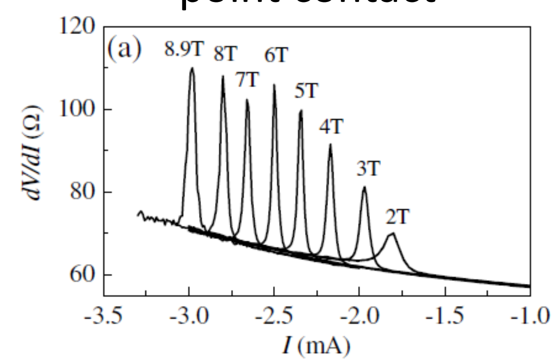
Current in-plane



nano-pillar



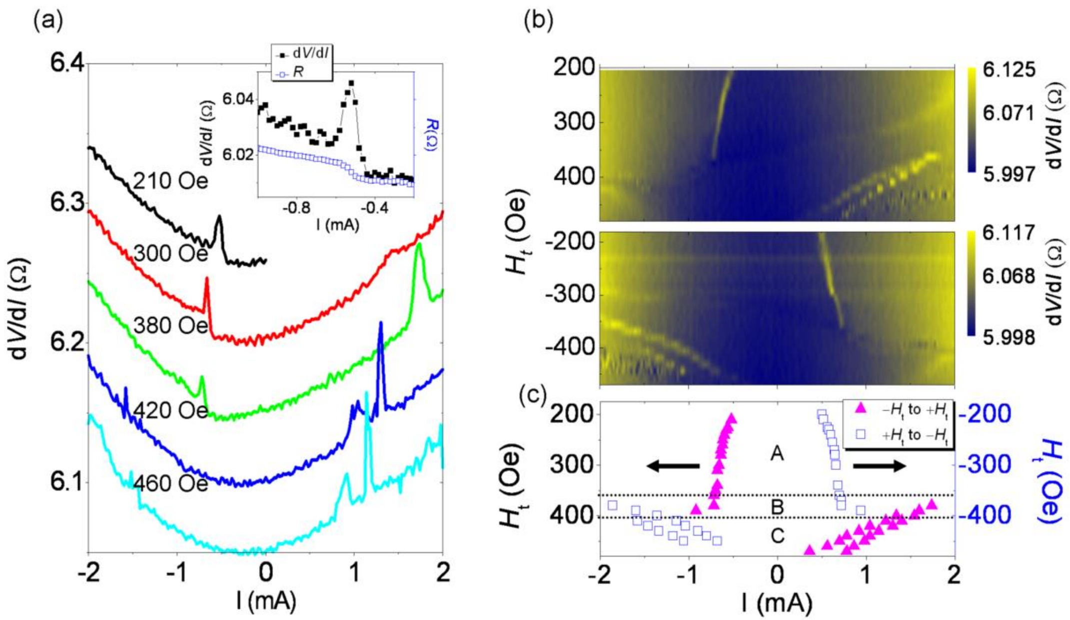
point contact

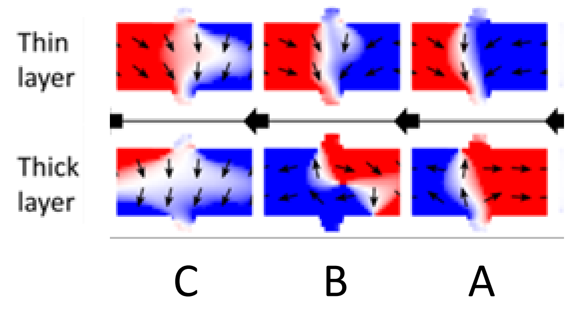


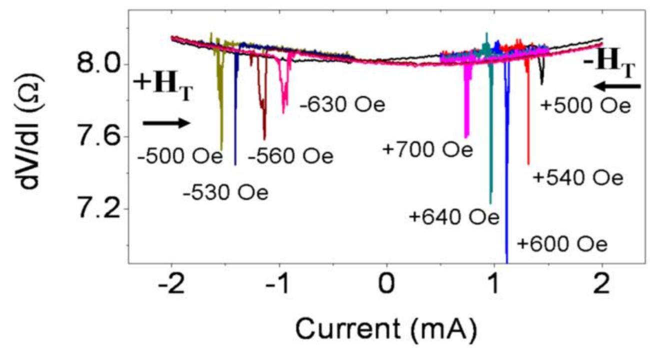
Science **307**, 228 (2005)
PRL **97**, 107204 (2006)

Measurement and simulation results

Reversible domain wall motion induced by dc current in NiFe/Cu/NiFe submicron wires



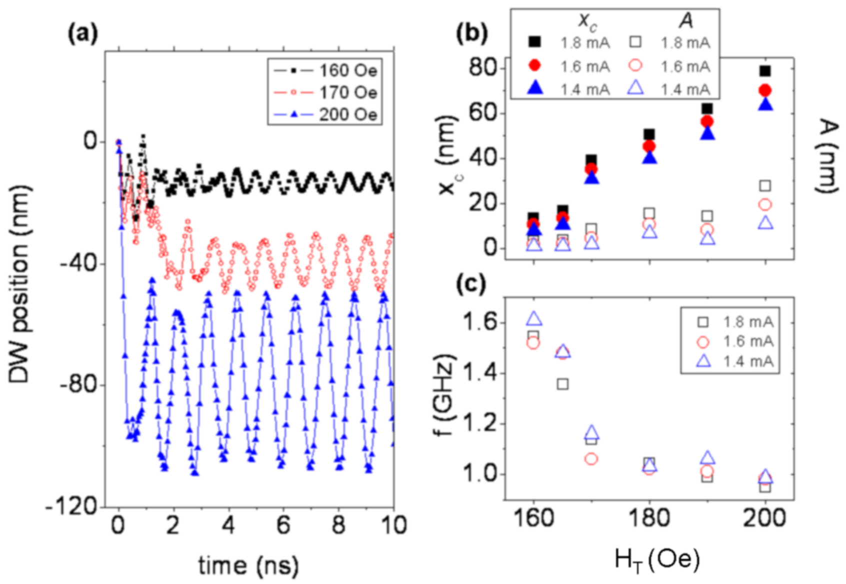




Differential resistance vs. current density at different external transverse fields H_t, enlarged in the inset for V/I vs. j at H_t = 210 Oe. (b) Map of dV/dI versus transverse field and dc current. (c) Critical current I_c vs. H_t.

Measurement and simulation results

Reversible domain wall motion induced by dc current in NiFe/Cu/NiFe submicron wires

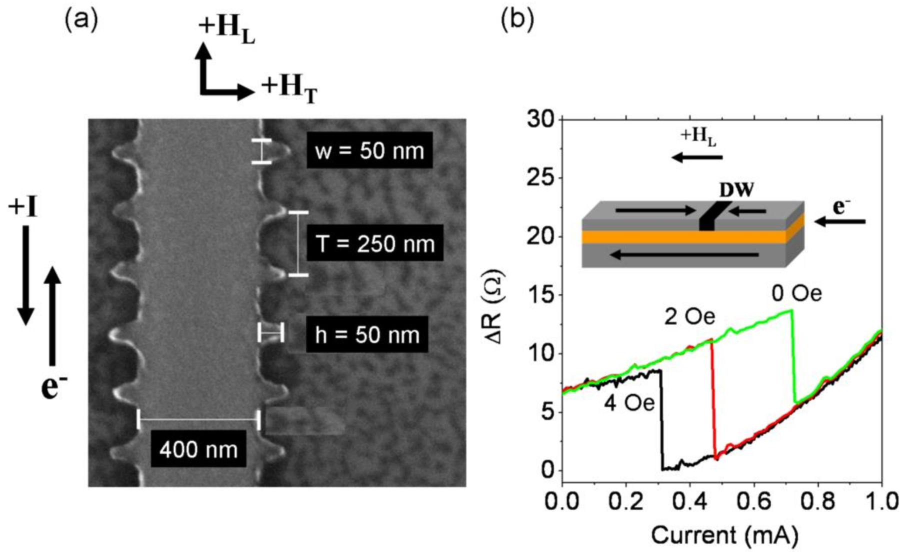


Simulation results of DW position as a function of time under fixed dc current density of 9.7×10^6 A/cm^2 with variation of external transverse field H_t. (b) central position x_c, amplitude A, and (c) frequency of the oscillator vs. H_t with different dc current.

Measurement and simulation results

Reversible domain wall motion induced by dc current in NiFe/Cu/NiFe submicron wires

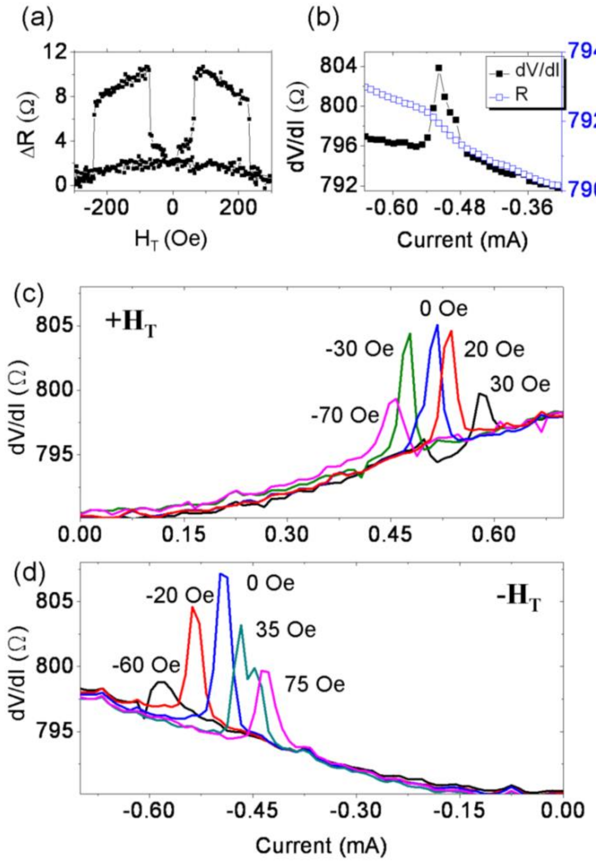
Series of submicron wires with serial DW traps of artificial symmetric protrusions

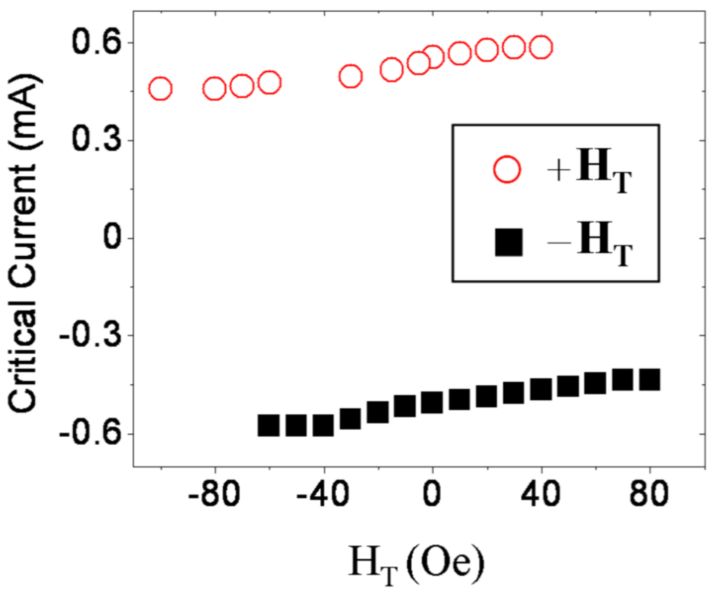


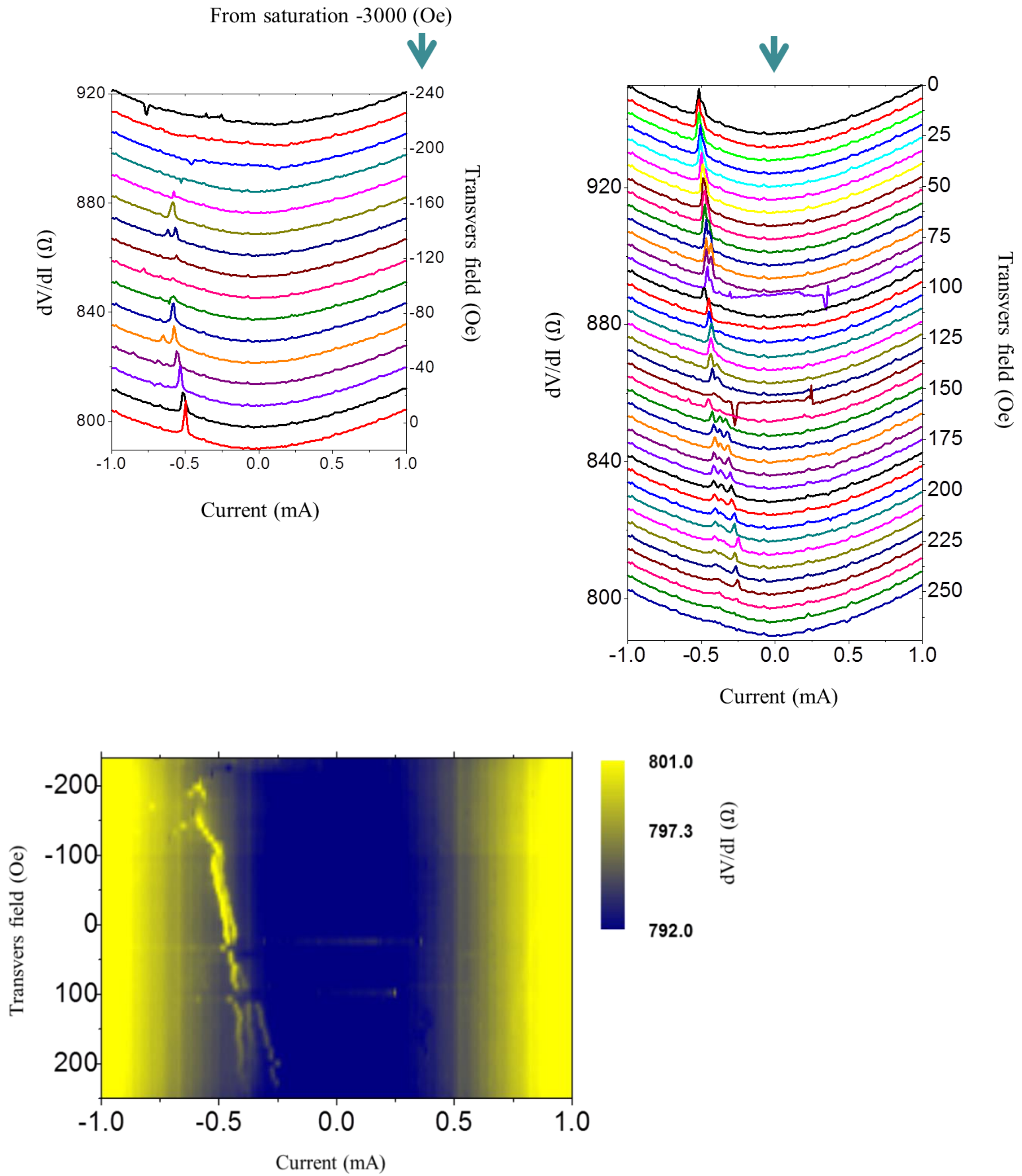
A Scanning electron microscope image of a typical serial-DW-trap sample with the protrusions 50 nm in width and height. The period was 250 nm on either side of the wire. Magnetic field and current directions are specified. (b) Schematic diagram of the sample and the irreversible resistance change from anti-parallel state to parallel state for H_L = 0 (green solid line), 2 (red dash line), and 4 (black dotted line) Oe.

Measurement and simulation results

Reversible domain wall motion induced by dc current in NiFe/Cu/NiFe submicron wires






From saturation -3000 (Oe)
dV/dI (Ω)
Transvers field (Oe)
Current (mA)
920
880
840
800
-240
-200
-160
-120
-80
-40
0
-1.0
-0.5
0.0
0.5
1.0
dV/dI (Ω)
Transvers field (Oe)
Current (mA)
920
880
840
800
0
25
50
75
100
125
150
175
200
225
250
-1.0
-0.5
0.0
0.5
1.0
Transvers field (Oe)
Current (mA)
-200
-100
0
100
200
-1.0
-0.5
0.0
0.5
1.0
801.0
797.3
792.0
dV/dI (Ω)

Summary

- DW oscillation with resonance frequency as high as 2.92 GHz and the resonance frequency can be tuned by the width of protrusion.

- The higher resonance frequency for the narrow trap is due to the steeper potential landscape which enhances the restoring force on the pinned DW.

- For the domain wall oscillations induced by injection of a dc current investigated, the observed peak in *dV/dI* associated with the reversible change of magnetoresistance is attributed to the reversible motion of the DW.

outline

- Giant Magnetoresistance, Tunneling Magnetoresistance
- Spin Transfer Torque
- Pure Spin current (no net charge current)
    - Spin Hall, Inverse Spin Hall effects
    - Spin Pumping effect
    - Spin Seebeck effect
- Micro and nano Magnetics

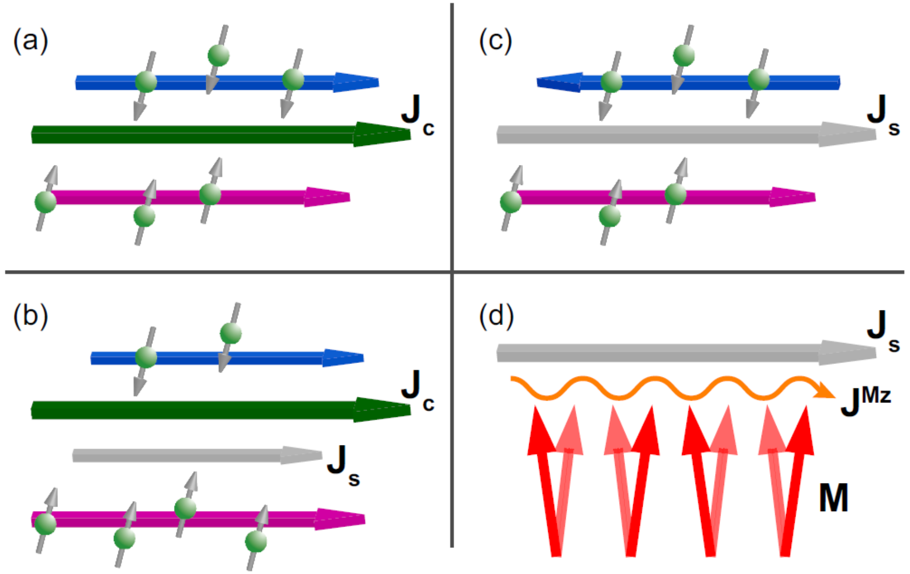
Spin polarized current vs pure spin current

Pure Spin Current

-- with no accompanying net charge current

- Theoretically
  - $J_S = \hat{s} \cdot \vec{v} \;\; \rightarrow J_S = \frac{d}{dt}(\hat{s} \cdot \vec{r})$

- Experimentally
  - Spin Hall, Inverse Spin Hall effects
  - Spin Pumping effect
  - Spin Seebeck effect



Spin Current

Proper Definition of Spin Current in Spin-Orbit Coupled Systems

Junren Shi,[1,2] Ping Zhang,[2,3] Di Xiao,[2] and Qian Niu[2]
[1]*Institute of Physics and ICQS, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*
[2]*Department of Physics, The University of Texas at Austin, Austin, Texas 78712, USA*
[3]*Institute of Applied Physics and Computational Mathematics, Beijing 100088, People's Republic of China*
(Received 19 April 2005; revised manuscript received 18 November 2005; published 24 February 2006)

The conventional definition of spin current is incomplete and unphysical in describing spin transport in systems with spin-orbit coupling. A proper and measurable spin current is established in this study, which fits well into the standard framework of near-equilibrium transport theory and has the desirable property to vanish in insulators with localized orbitals. Experimental implications of our theory are discussed.

$$J_S = \hat{s}\cdot\vec{v} \quad \rightarrow \quad J_S = \frac{d}{dt}(\hat{s}\cdot\vec{r}) = \hat{s}\cdot\vec{v} + \frac{d}{dt}\hat{s}\cdot\vec{r}$$

torque dipole term

1. spin current is not conserved
2. can even be finite in insulators with localized eigenstates only
3. not in conjugation with any mechanical or thermodynamic force, not fitted into the standard near-equilibrium transport theory

1. spin current conserved
2. vanishes identically in insulators with localized orbitals
3. in conjugation with a force given by the gradient of the Zeeman field or spin-dependent chemical potential

Spin Current

Advances in Physics
Vol. 59, No. 3, May–June 2010, 181–255

Taylor & Francis
Taylor & Francis Group

Spin currents and spin superfluidity

E.B. Sonin*

Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel

(*Received 3 September 2009; final version received 1 February 2010*)

The present review analyses and compares various types of dissipationless spin transport: (1) Superfluid transport, when the spin-current state is a metastable state (a local but not the absolute minimum in the parameter space). (2) Ballistic spin transport, when spin is transported without losses simply because the sources of dissipation are very weak. (3) Equilibrium spin currents, i.e. genuine persistent currents. (4) Spin currents in the spin Hall effect. Since superfluidity is frequently connected with Bose condensation, recent debates about magnon Bose condensation are also reviewed. For any type of spin currents simplest models were chosen for discussion in order to concentrate on concepts rather than the details of numerous models. The various hurdles on the way of using the concept of spin current (absence of the spin-conservation law, ambiguity of spin current definition, etc.) were analysed. The final conclusion is that the spin-current concept can be developed in a fully consistent manner, and is a useful language for the description of various phenomena in spin dynamics.

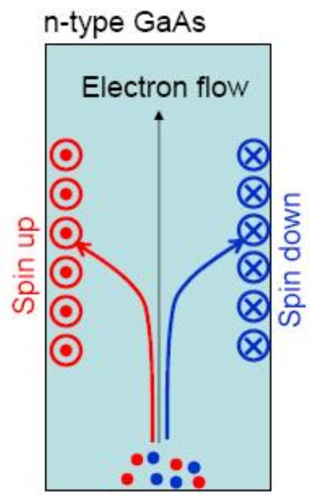
4. Conclusions

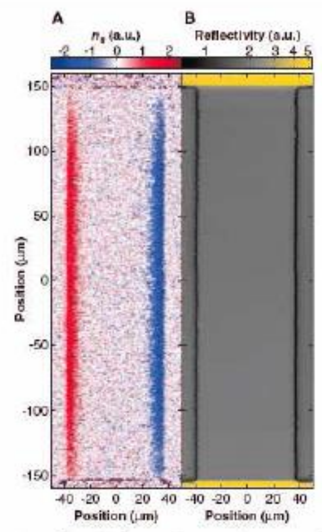
The present review focused on four types of dissipationless spin transport: (1) superfluid transport, when the spin-current state is a metastable state (a local but not the absolute minimum in the parameter space); (2) Ballistic spin transport, when spin is transported without losses simply because the sources of dissipation are very weak; (3) equilibrium spin currents, i.e. genuine persistent currents and (4) spin currents in the spin Hall effect. The dissipationless spin transport was a matter of debate for decades, though sometimes they were to some extent semantic. Therefore, it was important to analyse what physical phenomenon was hidden under this or that name remembering that any choice of terminology is inevitably subjective and is a matter of taste and convention. The various hurdles on the way of using the concept of spin current (absence of the spin-conservation law, ambiguity of spin current definition, etc.) were analysed. The final conclusion is that the spin-current concept can be developed in a fully consistent manner, though this is not an obligatory language of description: spin currents are equivalent to deformations of the spin structure, and one may describe the spin transport also in terms of deformations and spin stiffness.

The recent revival of interest to spin transport is motivated by the emerging of spintronics and high expectations of new applications based on spin manipulation. This is far beyond the scope of the present review, but hopefully the review could justify using of the spin-current language in numerous investigations of spin-dynamics problems, an important example of which is the spin Hall effect.

Spin Hall effect

Spin Hall Effect: Electron flow generates transverse spin current





SHE observed in GaAs using Kerr effect to measure spin

Kato et.al. (Awschalom), Science 306, 1910 (2004)

Now observed at room temperature in ZnSe

M. I. Dyakonov and V. I. Perel, *JETP* **13** 467 (1971)

J. E. Hirsch, *Phys. Rev. Lett.* **83** 1834 (1999)

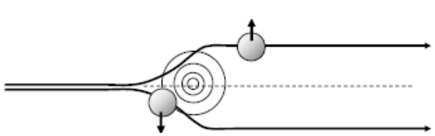
Guo *et al*, PRL **100** 096401 (2008)

The Intrinsic SHE is due to topological band structures

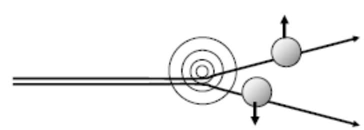
Berry curvature

$$\dot{\vec{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}} + \frac{e}{\hbar}\vec{E}\times\vec{\Omega}(\vec{k})$$

The extrinsic SHE is due to asymmetry in electron scattering for up and down spins. – spin dependent probability difference in the electron trajectories

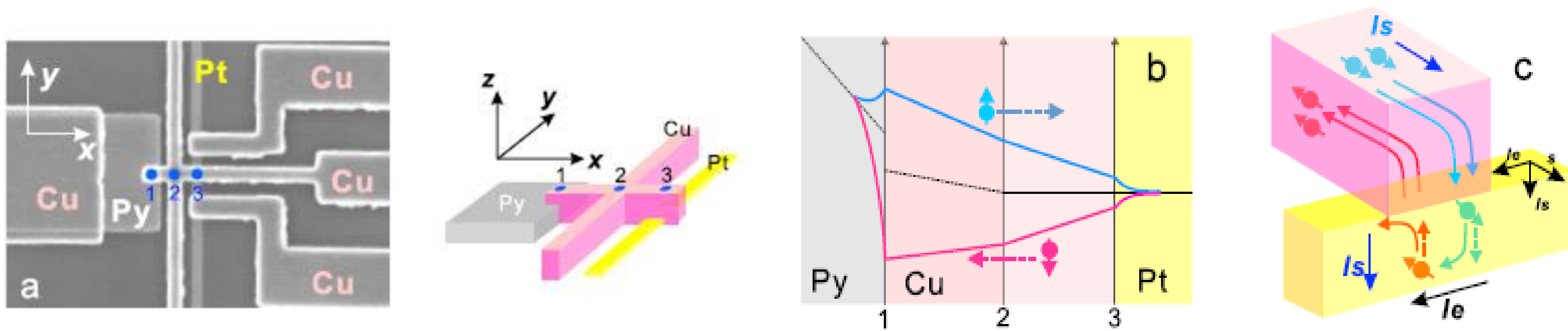


Side jump



Skew scattering

Inverse Spin Hall effect



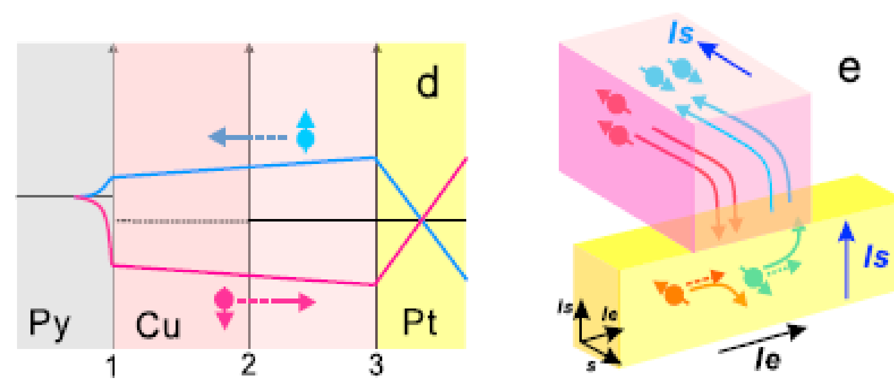


FIG. 1 (color online). (a) Scanning electron microscope (SEM) image of the fabricated spin Hall device together with a schematic illustration of the fabricated device. (b) Schematic spin dependent electrochemical potential map indicating spin accumulation in Cu and Pt induced by the spin injection from the Py pad. Dashed line represents the equilibrium position. (c) Schematic illustration of the charge accumulation process in the Pt wire, where I_S and I_e denote injected pure spin current and induced charge current, respectively. (d) Spin dependent electrochemical potential map for the charge to spin-current conversion and (e) corresponding schematic illustration.

Kimura *et al*, PRL **98**, 156601 (2007)

Guo *et al*, PRL **100** 096401 (2008)

Inverse Spin Hall effect : ISHE

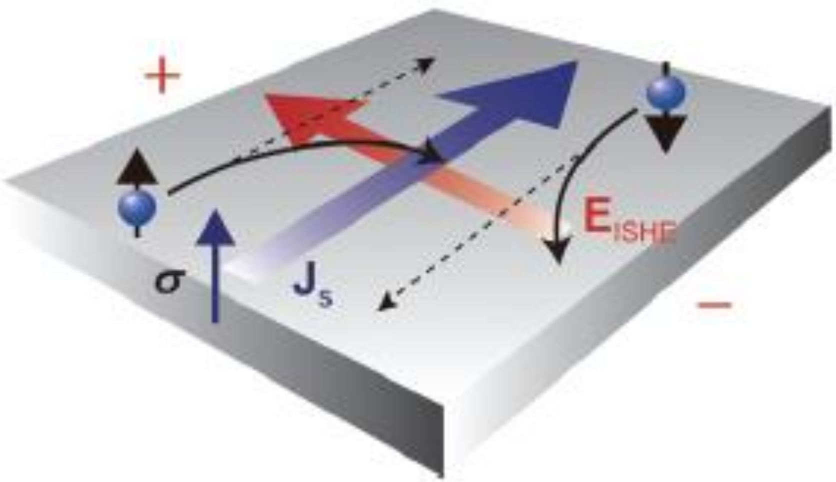
VOLUME 83, NUMBER 9 PHYSICAL REVIEW LETTERS 30 AUGUST 1999

Spin Hall Effect

J. E. Hirsch
Department of Physics, University of California, San Diego, La Jolla, California 92093-0319
(Received 24 February 1999)

It is proposed that when a charge current circulates in a paramagnetic metal a transverse spin imbalance will be generated, giving rise to a "spin Hall voltage." Similarly, it is proposed that when a spin current circulates a transverse charge imbalance will be generated, giving rise to a Hall voltage, in the absence of charge current and magnetic field. Based on these principles we propose an experiment to generate and detect a spin current in a paramagnetic metal.

ISHE: converts a spin current into an electric voltage



J. Appl. Phys. **109**, 103913 (2011)

SO-coupling bends the two electrons in the same direction → charge accumulation → E_{ISHE}.

$$\mathbf{E}_{\mathrm{ISHE}} \propto \mathbf{J}_s \times \boldsymbol{\sigma}$$

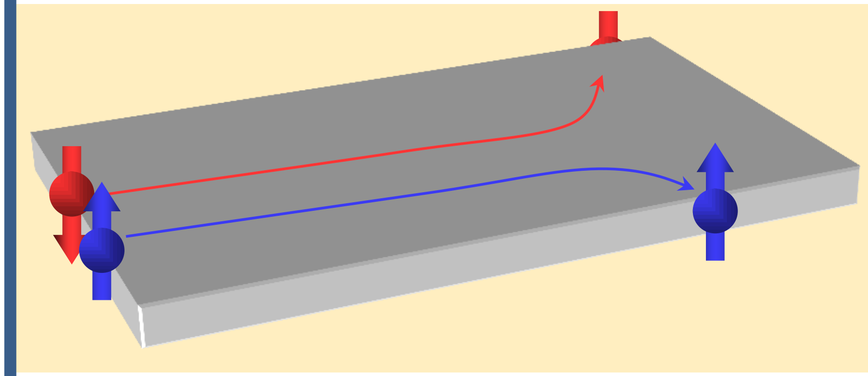
Js : spin current density

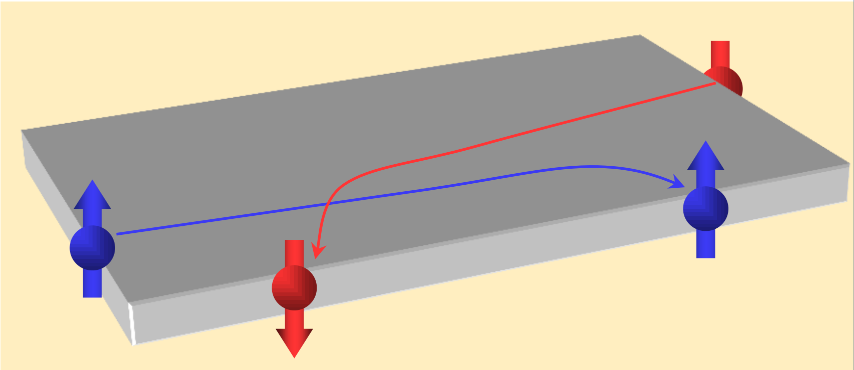
σ : direction of the spin-polarization vector of a spin current.

ISHE: Governed by spin-orbit coupling

SHE vs. ISHE

Spin Hall	Inverse Spin Hall
Charge Current	Spin Current
⇓	⇓
Transverse	Transverse
Spin Imbalance	Charge Imbalance





ISHE : direct & sensitive detection of a spin current !

Spin Hall Angle

$$\gamma = \frac{\sigma_{SH}}{\sigma_c}$$

σ_{SH} ← spin Hall conductivity

σ_c ← charge conductivity

stronger spin orbit interaction ➡ larger γ